COMPUTER-AIDED DESIGN OF A SIMPLE LOW-PAS AND HIGH-PASS MIXED LUMPED-DISTRIBUTED FILTER

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ABSTRACT

This report considers a mixed lumped-distributed filter consisting of two unit elements and three lumped reactive elements. It has been found that any analytical solution is highly difficult and therefore computer-aided design has to be resorted to. Both low-pass and high-pass filters are designed.

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CHAPTER I

1.1 General

Filters are generally clasified by their frequency response. The most commonly encountered ones are low-pass, band-pass, band-elimination and all-pass filters. However, depending upon the frequency range at which these filters are used, their constructional features differ. We shall briefly discuss the different types of filters that are commonly constructed in practice.

1.2 Lumped Passive Filters [1]

There are four types of passive filters namely RC, RL, LC and RLC type filters. All these can give low-pass, band-pass, high-pass and band-elimination filters; however RC and RL filter type transfer function have simple poles on negative real axis only and hence their response is less selective. Similarly, the response of the LC filter is restricted as poles of the transfer function can be on the imaginary axis only. The RLC filters, on the other hand, can give their poles anywhere in the left half of the S plane (excluding the imaginary axis) and hence the response can be made more selective.

We can now compare RC and LC types of filters:

(i) RC can be put in integrated circuit form, thus reducing its size considerably; LC normally, has a larger size because of the inductance, particularly at low frequencies.

(ii) Inductance is not an ideal element as there will always be a resistance associated with it.

(iii) Inductance requires shielding to prevent interference.

(iii) Inductance requires shielding to prevent radiation of magnetic fields and interference.

1.3 Active Filters [2]

On the contrary, the inductance can be avoided to simulate RLC filter action by using active elements like operational amplifiers. Active RC techniques for filter synthesis was considered an important technique. With the advent of integrated circuits, it has assumed wider importance. Some of the aspects of active RC filters is that it can not be used at frequencies higher than one MHZ approximately. However at low frequencies, this is highly useful.

We can compare the following characteristics for discrete elements against integrated circuits as used in active filtering:

(i) Accuracy

In discrete elements, accuracy is generally high because of the preselection of components, on the contrary, integrated circuits do not have absolute accuracy of discrete circuits because of the nature of production. However they can be highly matched thus improving the accuracy considerably.

(ii) Thermal Tracking

Compensation of temperature effects is one of the primary problems in monolithic structures, since a change in temperature leads to a uniform change in resistance values, and the frquency response is highly affected. In integrated circuits, special techniques result in obtaining low or zero sensitivity for changes due to temperatures.

(iii) Parasitics

Integrated circuits have reduced geometry and high packing densities, compared to discrete elements. This has the effect of reducing the parasitics considerably.

(iv) Economy

Because of mass-production techniques, integrated circuits cost less than discrete structures.

(v) Adjustments

Discrete structures are easier to adjust for a precise transfer function because of the availability of the individual components.

1.4 Digital Filters

The availability of fast, low cost digital integrated circuits has created a new interest in using digital filtering techniques in the design of signal processing systems. Digital filters can provide an outstanding degree of accuracy and stability especially at low frequencies where they can be miniaturised thus reducing size considerably with improved reliability. These filters are generally used:

(i) in biomedical applications, and

(ii) in frequencies of sub-audio range.

At high frequencies neither active nor digital filters can be used and hence distributed elements will be used.

1.5 Filters using Distributed Elements* [3]

A lossless transmission line or a wave guide is an example of a filter using distributed elements. The equivalent circuit is formed of an infinite number of sections as shown in figure 1.1 . Each section is acting like a filter, the cut off frequency of this

*RC distributed structures do not come in this category

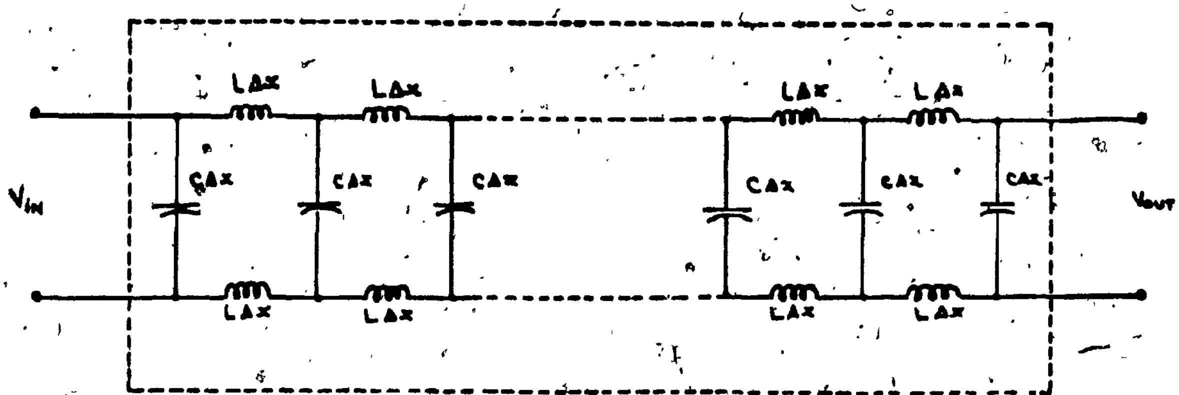

Figure 1.1 Equivalent circuit of a lossless transmission line.

filter depends on the physical dimensions of the element.

A sudden change in the physical dimensions of the transmission line give the same effect of adding a lumped element in the circuit, therefore creating a mixed lumped-distributed structure. In addition, lumped elements may be approximated by using structures which are built for microwave applications, A series inductance may be simulated by short sections of high impedance line (relatively thin rod surrouded by air dielectric). A shunt capacitance can be simulated by short sections of very low impedance line (consisting of a metal disc with a rim of dielectric). A typical low-pass filter can operate from a few hundred MHZ up to aroûnd ten MHZ.

The different ways of constructing them will be as follows:

(i) Mixed lumped-distributed structures may be constructed of alternate sections of high impedance (Z_o= 150 ohms) and low impedance (Z_o= 10 ohms) coaxial lines. The length of the high impedance section would be approximately one eight wave length. The length of the low impedance section is far shorter than one eight wave length.

(ii) Another method is to use a corrugated wave guide with uniform corrugations which can simulate the effect of low-pass filter, but due to the cut off frequency of the wave guide (λ_c = 2 a, where a is the width of the guide) the filter operates only at frequencies higher than f_c. A longitudinal slot through the corrugations help suppress the propagation of higher order modes.

We can now compare wave guide and coaxial line types of low-pass filters:

(i) Wave guide filters are larger and more expensive.

(ii) The stop bands in wave guides can not readily be made to be free of spurious responses to as high a frequency even for normal TE_{10}

mode of propagation.

(iii) There will be numerous spurious responses in the stop-band region for higher order modes, which are easily excited at frequencies above the normal TE_{10} operating range of the wave guide.

1.6 Lumped-Distributed Networks [4]

As discussed earlier, at higher frequencies filters can be built of sections of transmission lines of different length, cross sections and lumped elements. This gives rise to mixed lumped-distributed structures. Also transmission lines or wave guides can be used between stages of circuits containing transistors or semiconductors. Also transmission lines may inevitably have lumped discontinuities. All these can be modelled by lumped-distributed structures. In addition the following may be claimed as advantages of such mixed lumped-distributed structures.

(i) Terminations in general are not purely resistive. A mixed lumped-distributed filter structure takes into account the presence of the parasitic elements.

(ii) Whenever combined filtering and impedance transformation is desirable such structures can be used. A mixed lumped-distributed structure using a quarter wave transformer is a common example.

(iii) In the case of comb-line filters, lumped capacitive coupling at the input and output reduces the size of the filter by eliminating the transmission line matched section.

(iv) In the case of cascaded U.E.'s filters, the number of U.E.'s can be reduced from (2n+1) to n by separating them by (n+1) lumped capacitors.

It is known that the realization techniques available in the lumped network theory can not be directly applied to realize mixed lumped-distributed structures due to the transcendental nature of the network functions. It is known that the driving point immittance of a network consisting of lumped, linear, finite and passive elements can be expressed as rational functions of the complex frequency variable S as:

$$Z(S) = \frac{P(S)}{Q(S)} \tag{1.1}$$

Where both P(S) and Q(S) are Hurwitz polynomials; That is, they have their zeroes only in the left half of the S plane. It is also noted that only positive real functions can be realized as the input immittance of a lumped, linear, finite and passive network.

The rational function is positive real iff

(i) F(S) is real, when S is real, and

(ii) Re $F(S) > 0$ for Re $S > 0$.

However, when a network consists of distributed elements also like transmission lines, wave guides etc. , the input immittance of that mixed network need not be a rational function of S, but can be modified to become a two-variable or a multivariable rational function as shown by the following example[4]:

Figure 1.2 shows a network consisting of five sections, three lumped elements and two sections of transmission lines. This network is frequently used in super-high frequency ranges. Looking at X-X and using the chain matrix, we find that:

$$Z(S) = \frac{A_t}{C_t} \tag{1.2}$$

Where A_t and C_t are the overall chain matrix parameters. The chain

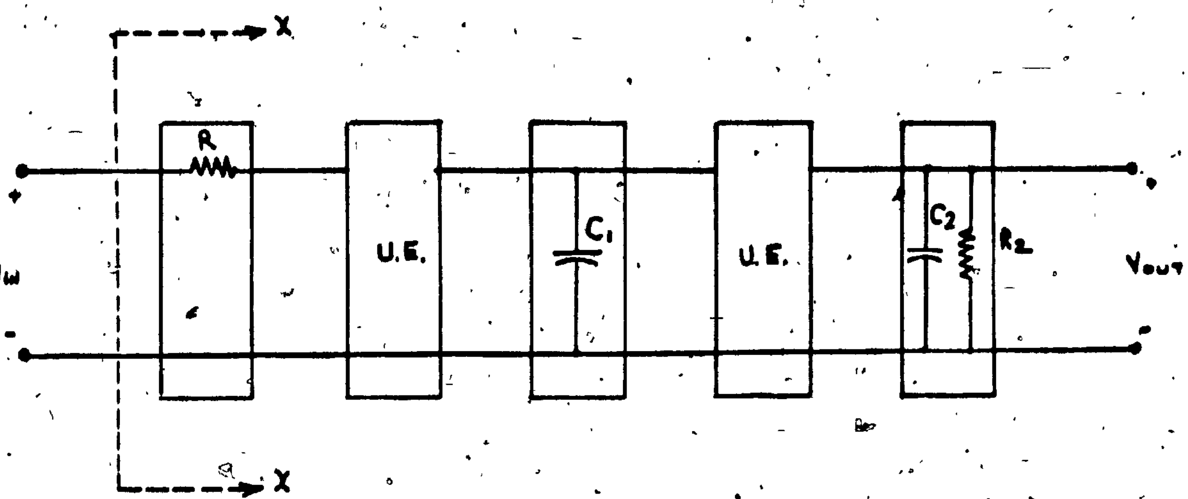

Figure 1.2 Mixed lumped-distributed network formed of three lumped elements and two distributed elements.

matrix of the transmission line is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \text{Cosh } S\tau & Z_o \text{ Sinh } S\tau \\ \dfrac{\text{Sinh } S\tau}{Z_o} & \text{Cosh } S\tau \end{bmatrix} \tag{1.3}$$

Where Z_o is the characteristic impedance and τ is the time delay of the transmission line.

Defining:

$$P_1 = S$$

$$\text{Sinh } S = \frac{P_2}{\sqrt{1-P_2^2}}$$

$$\text{Cosh} = \frac{1}{\sqrt{1-P_2^2}} \tag{1.4}$$

We express the chain matrix of the transmission line as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1-P_2^2}} \begin{bmatrix} 1 & Z_o P_2 \\ \dfrac{P_2}{Z_o} & 1 \end{bmatrix} \tag{1.5}$$

The overall chain matrix is:

$$\begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{1 - P_2^2}} \begin{bmatrix} 1 & Z_o P_2 \\ \frac{P_2}{Z_o} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ SC_1 & 1 \end{bmatrix} \frac{1}{\sqrt{1 - P_2^2}}$$

$$\begin{bmatrix} 1 & Z_o P_2 \\ \frac{P_2}{Z_0} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (SC_2 + \frac{1}{R_2}) & 1 \end{bmatrix} \tag{1.6}$$

From which we obtain:

$$Z(S) = \frac{1}{\sqrt{1 - P_2^2}} \quad \frac{X - R_1 \quad Z_1}{Z_1} \tag{1.7}$$

Where:

$$X = 1 + P_1^2 P_2^2 Z_o^2 C_1^2 + 3 Z_o C_1 P_1 P_2 + \frac{2}{R_2} Z_o P_2 + \frac{C_1 Z_o^2 P_2^2}{R_2} + P_2^2 + 1 \tag{1.8}$$

and

$$Z_1 = Z_o C_1^2 P_1^2 P_2 + C_1 P_1 P_2^2 + 2 C_1 P_1 + \frac{Z_o C_1 P_1 P_2}{R_2} + P_2^2 + 1 \tag{1.9}$$

We note that equation (1.7) is a rational function of the two-variables namely P_1 and P_2, but is not a rational function of S. Many more examples can be quoted.

There exist basically two approaches to realize mixed lumped-distributed structures, namely the single variable and the multivariable. The sigle variable approach deals directly with transcendental functions. In the multivariable approach, the transcendental function of S is converted into several variables S and P_i. This has been discussed earlier. However in the single variable approach, the network functions for a certain class of distributed structures can be transformed into rational functions of the transformed variable by using suitable transformations. One transformation is to introduce $P=\tanh S\tau$ (where τ is the time delay of the transmission line as seen earlier). Then the input immittance of a network consisting of finite number of lumped resistors, transformers and U.E.'s of commensurate lengths is a positive real function in P. Another approach consists in expressing the input impedance for a resistively terminated cascade of commensurate U.E.'s as a ratio of sums of exponentials, the coefficient of the exponentials being real constants. Another approach is the multivariable one. Some accepted definitions of multivariable functions are given below:

Definition 1.1

A rational function $F(P_1, P_2)$ is called a two-bariable positive real function (T.P.R.F.) when the following conditions are satisfied:

(i) $F(P_1, P_2)$ is real, when P_1 and P_2 are real,

(ii) $\operatorname{Re} F(P_1, P_2) > 0$, whenever $\operatorname{Re} P_1 > 0$, $\operatorname{Re} P_2 > 0$

Definition 1.2

A rational function $F(P_1, P_2)$ is called a two-variable reactance function (T.R.F.) when the following conditions are satisfied:

(i) $F(P_1, P_2)$ is a P.R.F.

(ii) $F(P_1, P_2) = -F(-P_1, -P_2)$

Definition 1.3

A polynomial $D(P_1, P_2)$ is called a two-variable hurwitz polynomial in the narrow sense (T.H.P.N.), if it has no zeroes in the regions:

(i) Re $P_1 > 0$, Re $P_2 = 0$,

(ii) Re $P_1 = 0$, Re $P_2 > 0$ and

(iii) Re $P_1 > 0$, Re $P_2 > 0$

Definition 1.4

A polynomial $D(P_1, P_2)$ is a two-variable hurwitz polynomial in the broad sense (T.H.P.B.), if it has no zeroes in the open polydomain Re $P_i > 0$, $i=1,2$ and those zeroes for the Re $P_i = 0$ must be simple.

It can be seen that these definitions can be considered as logical extensions of those in the single-variable case.

1.6.1 Multivariable Array in Ladder Networks

Any single-variable reactance function can be always synthesized as a low-pass ladder network by a continued fraction expansion. However every single-variable positive-real function can not be realised by a continued fraction expansion. Similarly, since a two-variable reactance function can be considered as a generalization of of sigle-variable positive-real function[5], we conclude that not all two-variable reactance functions are realizable by continued fraction expansion as ladder networks. Ramachandran and Rao[6] have derived conditions under which a multivariable reactance function (M.R.F.) can be expanded into continued fractions thereby realizing it as a low-pass ladder network. By means of transformations, several other ladder networks are obtainable. Specifically[6] have shown that from the given (M.R.F.) a multivariable array can be constructed from which it can be found when a given M.R.F. is realizable as a low-pass ladder network. The applications of such low-pass ladders are also discussed and they are:

(i) Networks with stubs and lumped elements where the inductors and the capacitors can, respectively, be replaced by non commensurate short circuited and open-circuited stubs.

(ii) Cascade of U.E.'s and lumped elements.

It has been shown that there exists an equivalence between a two-variable low-pass ladder network and a cascade connection of lossless uniform transmission lines (U.E.'s) separated by series lumped inductors on one side and shunt lumped capacitors on the other side as shown in figure 1.3 .

1.7 Scope of the report

To the best of the author's knowledge, even though the conditions of physical realizability have been established, such structures have not been used to design low-pass and high-pass filters. This report discusses a computer-aided approach to acheive the above aim, when the filter sections consist of three lumped elements and two unit elements connected as shown in figure 1.4 .

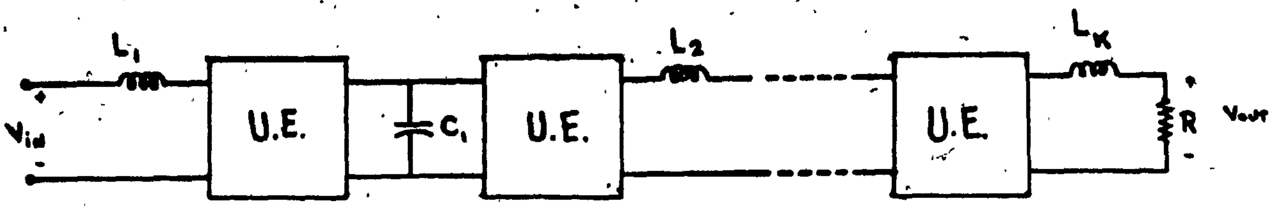

Figure 1.3 Network consisting of cascade connection of lossless uniform transmission lines separated by series inductors on one side and shunt lumped capacitors on the other side.

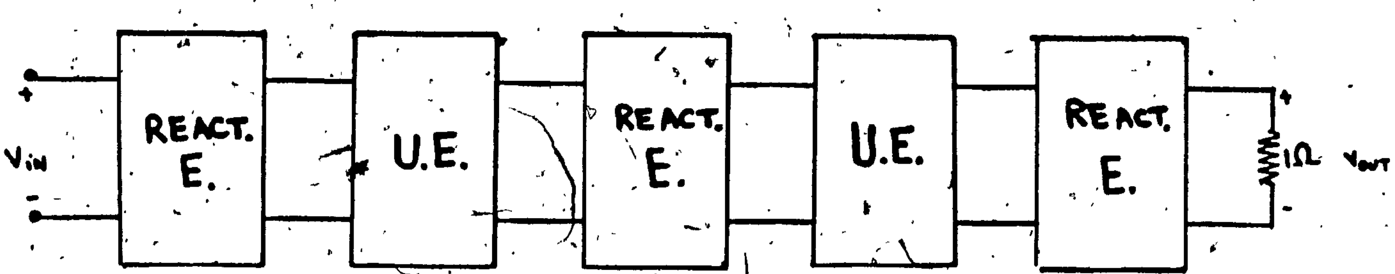

Figure 1.4 Mixed lumped-distributed network consisting of three lumped reactive elements and two unit elements. The network is terminated in one ohm.

CHAPTER II

A LOW-PASS MIXED LUMPED-DISTRIBUTED FILTER

2.1 Introduction

In this chapter, we shall consider the design of a low-pass filter obtained from the configuration shown in figure 1.4 . The lumped elements are L_1, C and L_2 , the distributed network consists of two unit elements with characteristic impedance Z_o=1 ohm. The entire network is terminated in a one ohm resistance. Before any attempt is made to design the fiter, it is essential to find its transfer function.

2.2 Analysis of the Network

The considered filter is shown in figure 2.1 . In order to find the transfer function, the chain matrix method is used[7]. We have, for the lumped elements :

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & SL_1 \\ 0 & 1 \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ SC & 1 \end{bmatrix} \tag{2.2}$$

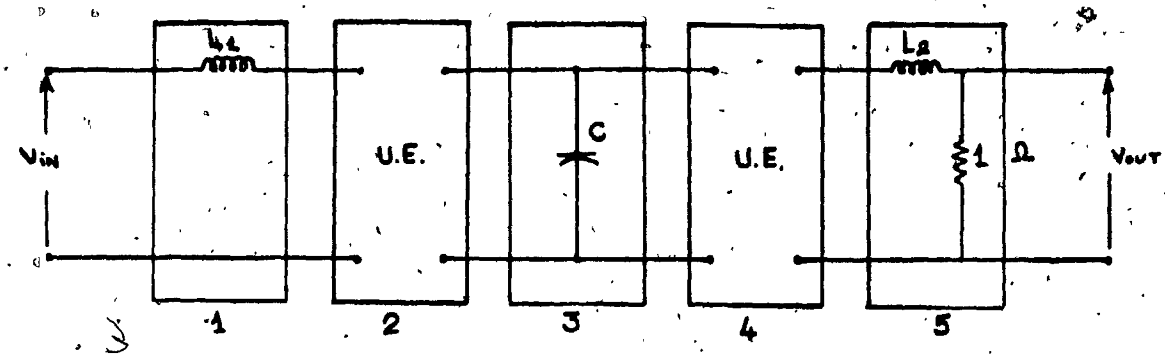

Figure 2.1 Network consisting of five sections. Two inductive reactances, one capacitive reactance and two unit elements. The network is terminated in a one ohm resistance.

and

$$\begin{bmatrix} A_5 & B_5 \\ & \\ C_5 & D_5 \end{bmatrix} = \begin{bmatrix} 1+SL_2 & SL_2 \\ & \\ 1 & 1 \end{bmatrix} \qquad (2.3)$$

and for the distributed elements :

$$\begin{bmatrix} A_2 & B_2 \\ & \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} A_4 & B_4 \\ & \\ C_4 & D_4 \end{bmatrix} = \begin{bmatrix} \text{Cosh } \gamma l & \text{Sinh } \gamma l \\ & \\ \text{Sinh } \gamma l & \text{Cosh } \gamma l \end{bmatrix} \qquad (2.4)$$

Where γ is the propagation constant of the U.E.

l is the electrical length of the U.E.

The Overall chain matrix is obtained by multiplying the five successive matrices. The transfer function of the network is the inverse of the overall A parameter. Therefore :

$$\begin{aligned} A_{overall} &= A_1 A_2 A_3 A_4 A_5 + A_1 B_2 C_3 A_4 A_5 \\ &+ A_1 A_2 A_3 B_4 C_5 + A_1 B_2 C_3 B_4 C_5 \\ &+ A_1 A_2 B_3 C_4 A_5 + A_1 B_2 D_3 C_4 A_5 \end{aligned}$$

$$+A_1A_2B_3D_4A_5+A_1B_2D_3D_4C_5$$
$$+B_1C_2A_3B_4C_5+B_1D_2C_3A_4A_5$$
$$+B_1C_2A_3A_4A_5+B_1D_2C_3B_4C_5$$
$$+B_1C_2B_3C_4A_5+B_1D_2D_3C_4A_5$$
$$+B_1C_2B_3D_4C_5+B_1D_2D_3D_4C_5 \qquad (2.5)$$

Substituting each parameter in equation (2.4) and considering $\gamma l = S\tau$, where τ is the time delay, we get:

$$A_{overall} = \mathrm{Cosh}^2\, S\tau\left((1+SL_2)+S^2L_1C+S^3L_1L_2C+SL_1 \right) + \mathrm{Sinh}^2\, S\tau(1+SC+SL_2+SL_1) + \mathrm{Sinh}\, S\tau\, \mathrm{Cosh}\, S\tau\, (2+S(2L_1+L_2+C) + S^2(L_1C+2L_1L_2)\,) \qquad (2.6)$$

Knowing that:

$$\mathrm{Cosh}^2\, S\tau = \frac{1}{2} + \frac{1}{2}\,\mathrm{Cos}\,(2\,w\tau),\ \text{and}$$
$$\mathrm{Sinh}^2\, S\tau = \frac{-1}{2} + \frac{1}{2}\,\mathrm{Cos}\,(2\,w\tau),\ \text{and}$$
$$\mathrm{Sinh}\, S\tau\ \mathrm{Cosh}\, S\tau = \frac{j}{2}\,\mathrm{Sin}\,(2w\tau). \qquad (2.7)$$

Therefore:

$$A_{overall} = F + j\,G \qquad (2.8)$$

and

$$\text{Modulus } T_V = \frac{F}{\sqrt{F^2 + G^2}} \tag{2.9}$$

Where:

$$F = -\frac{1}{2} w^2 L_1 C + \frac{1}{2} \operatorname{Cos} (2w\tau) (2 - w^2 L_1 C)$$

$$- \frac{1}{2} \operatorname{Sin} (2w\tau) (w(2L_1 + L_2 + C)) \tag{2.10}$$

and

$$G = -\frac{1}{2} wC - \frac{1}{2} w^3 L_1 L_2 C$$

$$+ \frac{1}{2} \operatorname{Cos} (2w\tau) (w(2L_1 + 2L_2 + C) - w^3 L_1 L_2 C)$$

$$+ \frac{1}{2} \operatorname{Sin} (2w\tau) (2 - w(L_1 C + 2L_1 L_2)) \tag{2.11}$$

Substituting in equation (2.9), we get:

$$T_V = 1/\text{SQRT} (1 + w^2 (L_1^2 + \frac{5}{8} L_2^2 + \frac{1}{2} L_1 L_2 + \frac{3}{4} L_1 C + \frac{1}{8} C^2)$$

$$+ w^4 (\frac{1}{2} L_1^2 L_2^2 + \frac{3}{4} L_1^2 C - \frac{1}{2} L_1 L_2^2 C - \frac{3}{4} L_1 L_2 C^2)$$

$$+ w^6 (\frac{5}{8} L_1^2 L_2^2 C^2)$$

$$+ \operatorname{Cos} (2w\tau) (-w^2 (2L_1 C + L_2 C + \frac{1}{2} C^2)$$

$$- w^4 (L_1^2 L_2 C + L_1 L_2^2 C - \frac{1}{2} L_1^2 C^2)$$

$$+ w^6 (\frac{1}{2} L_1^2 L_2^2 C^2))$$

$$+ \operatorname{Sin}(2w\tau)\,(-wC$$

$$+w^{3}\left(\frac{2}{3}L_{1}C^{2} + \frac{1}{2}L_{1}L_{2}C + \frac{1}{2}L_{1}C^{2}\right)$$

$$+w^{5}\left(\frac{1}{2}L_{1}^{2}L_{2}C^{2} + L_{1}^{2}L_{2}^{2}C\right)$$

$$+ \operatorname{Cos}(4w\tau)\,(w^{2}\left(\frac{2}{3}L_{2}^{2} + \frac{1}{4}L_{2}C + \frac{3}{2}L_{1}L_{2}\right)$$

$$- w^{4}\left(\frac{1}{2}L_{1}^{2}L_{2}^{2} + \frac{1}{4}L_{1}L_{2}C^{2} + \frac{1}{2}L_{1}L_{2}^{2}C + L_{1}^{2}L_{2}C\right)$$

$$+ w^{6}\left(\frac{1}{8}L_{1}^{2}L_{2}^{2}C^{2}\right))$$

$$+ \operatorname{Sin}(4w\tau)\,(w\left(\frac{1}{2}L_{2}\right)$$

$$- w^{3}\left(\frac{5}{4}L_{1}L_{2}C + \frac{1}{2}L_{1}^{2}L_{2} + \frac{1}{2}L_{1}L_{2}^{2}\right)) \qquad (2.12)$$

In order to have a low-pass filter, the following condition must be fulfilled:

$$\left|T_{v}\right| \leqslant 1 \qquad (2.13)$$

or

$$w(-C \operatorname{Sin}(2w\tau) + \frac{1}{2}L_{2} \operatorname{Sin}(4w\tau))$$

$$+w^{2}\left(\left(L_{1}^{2} + \frac{5}{8}L_{2}^{2} + \frac{1}{8}C^{2} + \frac{1}{2}L_{1}L_{2} + \frac{3}{4}L_{1}C\right)\right.$$

$$- \cos(2w\tau)\left(2L_1C+L_2C+\frac{3}{4}L_1C\right)$$

$$+\cos(4w\tau)\left(\frac{3}{8}L_2^2+\frac{1}{4}L_2C+\frac{3}{2}L_1L_2\right)\Big)$$

$$+w^3\left(\sin(2w\tau)\left(\frac{3}{2}L_1C^2+\frac{1}{2}L_1L_2C+\frac{1}{2}L_1C^2\right)\right.$$

$$\left.-\sin(4w\tau)\left(\frac{5}{4}L_1L_2C+\frac{1}{2}L_1L_2^2+\frac{1}{2}L_1L_2^2\right)\right)$$

$$+w^4\Big(\left(\frac{1}{2}L_1^2L_2^2+\frac{3}{4}L_1^2C^2-\frac{1}{2}L_1L_2^2C-\frac{3}{4}L_1L_2C^2\right)$$

$$-\cos(2w\tau)\left(L_1^2L_2C+L_1L_2^2C-\frac{1}{2}L_1^2C^2\right)$$

$$-\left(\frac{1}{2}L_1^2L_2^2+\frac{1}{4}L_1L_2C^2+\frac{1}{2}L_1L_2^2C+L_1L_2^2C\right)\cos(4w\tau)\Big)$$

$$+w^5\left(\sin(2w\tau)\left(\frac{1}{2}L_1L_2^2C^2+L_1^2L_2^2C\right)\right)$$

$$+w^6\left(\frac{5}{8}L_1^2L_2^2C^2+\cos(2w\tau)\left(\frac{1}{2}L_1^2L_2^2C^2\right)+\cos(4w\tau)\left(\frac{1}{8}L_1^2L_2^2C^2\right)\right)$$

$$\leqslant 0 \qquad (2.14)$$

Analytical solution of this equation is difficult and no direct method can be applied to find its roots.

In order to simplify equation (2.14), and since we are working at low frequencies, we can substitute $\cos(2w\tau)$ by 1, and $\sin(2w\tau)$ by $2w\tau$ in equations (2.10) and (2.11). Therefore:

$$\left|T_V\right| = 1/\,\mathrm{SQRT}\left(\left(1-w^2(L_1C+2L_1\tau+L_2\tau+C\tau)\right)^2 + \left(2w\tau+wL_1+wL_2-w^3(L_1L_2C+L_1C\tau+2L_1L_2\tau)\right)^2 \right) \tag{2.15}$$

or

$$\begin{aligned} &w^2(2\tau^2+L_1^2+L_2^2+4L_1\tau+4L_2\tau+2L_1L_2)+w^4(L_1^2C^2+4L_1^2\tau^2+L_2^2\tau^2+C^2\tau^2+2L_1^2C \\ &\quad -2L_1L_2C+2L_1C^2-4L_1L_2^2\tau^2+2L_2C^2-2L_1^2L_2C-4L_1^2L_2\tau-2L_1L_2^2C) \\ &+w^6(L_1^2L_2^2C^2+L_1^2C^2\tau^2+4L_1^2L_2^2\tau^2+2L_1^2L_2C^2\tau+4L_1^2L_2^2C\tau+4L_1^2L_2C\tau^2) \leqslant 0 \end{aligned} \tag{2.16}$$

This expression is complicated in view of the fact that there are four variables to adjust. We can consider some cases like letting $L_1=L_2=KC$ in which the lumped section is aconstant K filter, equation (2.16) becomes:

$$\begin{aligned} &4\tau^2+4C^2K^2+8CK\tau+w^2C^2\left((\tau^2+3\tau^2K^2)+C(2\tau K-8\tau K^3)-4C^2K^3\right) \\ &+w^4C^4(C^2K^4+\tau^2K^2+4\tau^2K^4+2C\tau K^3+4C\tau K^4+4\tau^2K^3) \leqslant 0 \end{aligned} \tag{2.17}$$

Equation (2.17) is quadratic in w^2; its descriminitor $b^2-4ac \geqslant 0$, and

$a > 0$. (2.18)

Where:

$$a = C^6K^4 + C^4\tau^2K^2 + 4C^4\tau^2K^4 + 2C^5K^3 + 4C^5\tau K^4 + 4C^4\tau^2K^3\text{, and}$$

$$b = C^2(\tau^2 + 3\tau^2K^2) + C^3(2\tau K - 8\tau K^3) - 4C^4K^3)\text{, and}$$

$$c = 4\tau^2 + 4C^2K^2 + 8C\tau K. \tag{2.19}$$

or

$$C^4\tau^4(1+22K^2+K^3+73K^4+64K^6) + C^5\tau^3(4K+60K^3+192K^4+80K^5)$$

$$+C^6\tau^2(-8K^3+96K^4+168K^5) + C^7\tau(32K^3-16K^4+32K^5-64K^6) > 0 \tag{2.20}$$

Since $C \neq 0$, equation (2.20) can be simplified to:

$$\tau^4(1+22K^2+K^3+73K^4+64K^6) + C\tau^3(4K+60K^3+192K^4+80K^5)$$

$$+C^2\tau^2(-8K^3+96K^4+168K^5) + C^3\tau(32K^3-16K^4+32K^5-64K^6) > 0 \tag{2.21}$$

The analytical solution of equation(2.21) is still fifficult, although we have approximated the harmonic terms and considered a special case of filters.

As the number of variables even for this network with small number of sections will become quite large, analytical solution becomes very difficult. Hence a computer-aided design seems to be well suited to determine the values of different elements.

Now we can consider some special cases as well as the general case.

2.2.1 The Lumped Section form a Constant K Filter

For this case, we have the relationship $L_1 = L_2 = KC$. Hence the transfer function $|T_V|$ becomes:

$$|T_V| = 1/SQRT(1+Sin(2w\tau)+0.5\,wC(-1+(4K+1)\,Cos(2w\tau) -(3K+1)\,Sin(2w\tau))$$
$$-0.5\,w^2C^2K(1+Cos(2w\tau)+(2k+1)\,Sin(2w\tau))$$
$$-0.5\,w^3C^3K^2(1+Cos(2w\tau)) \qquad (2.22)$$

The computer program for this case is given in Table 2.1 . We see that there are three variables to adjust namely K,C and τ . This yields families of curves which are shown in figures 2.4.1 to 2.4.15 .

Figure 2.2.1 gives the response for small K (0.1) and small C (0.1), we notice that oscillations occur at frequencies depending on τ . This shows the effect of the time delay of the ditributed section. Since a mismatch can occur at the input and output of the transmission line, reflections appear.

As τ is increased the magnetude of the reflections becomes higher and higher oscillations take place. Also the frequency at which the oscillations occur increases with τ .

Figure 2.2.2 shows the effect of increasing C for the same value of the constant K. For C=1 and τ =0.8 a very high peak occurs at w=3.75, in addition to other scillations. It is noticed that the peaks tend to move to the left as C is increased keeping τ fixed. This is due to the property of low-pass filtering since a faster cut-off occurs with increased C.

Figure 2.2.3 shows the response when K is increased (K=0.5).

Oscillations continue to appear and the maximum shoots to 28.

Figure 2.2.4 shows the effect of increasing the value of the capacitance (C=1). For the value of K=0.5 and τ=4.5, the cut-off frequency is decreased and oscillations move to the left.

Figure 2.2.5 shows a typical low-pass filter. Comparing the two graphs for K=0.5, C=0.2 , τ=0.1 and K=0.5, C=0.4, τ=0.1 we conclude that cut-off decreases as C increases.

Figure 2.2.6 shows the effect of increasing for fixed values of C and K (C=0.5, K=0.5). It is noticed that cut-off is slightly decreased with the increase of τ. A very sharp cut-off appears for C=23.5

Figure 2.2.7 gives a family of curves for K=0.6 and τ=1, the same property discussed earlier is shown.

Figure 2.2.8 shows the response for higher K (K=1.5), a peak appears at w=1.7.

Figure 2.2.9 shows the effect of increasing τ (τ=0.2 to 1). Cut-off is displaced to the left.

Figure 2.2.10 gives a family of curves for K=1.5, τ=0.2 and C varying. Cut-off moves to the left as C is increased.

Figure 2.2.11 gives another family of curves for K=1.5 and C = 1.5, Different values of τ are shown. In general an increase in the value of τ decreases the cut-off frequency.

Figure 2.2.12 shows the effect of increasing the value of the constant K (K=1.0 to 1.5) for the same values of C and τ (C=1, τ=1). An increase in the value of K decreases the magnitude of oscillations in addition to the decrease of the cut-off frequency.

Figures 2.2.13 and 2.2.14 are a family of curves for K=2 and K=4, τ=1.0 . Increasing C decreases the cut-off frequency as seen

```
      PROGRAM FAROUK (INPUT,OUTPUT)
C     STUDY OF A L.P.F. OF MIXED LUMPED DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=0.1
      C=0.1
      Y=0.0
33    X(1)=0.0
      DO 100 I=1,100
      R=2.*X(I)*Y
      B=COS(R)
      D=SIN(R)
      E=-0.5*X(I)**2*C**2
      F=-0.5*X(I)*C
      G=-0.5*X(I)**3*C**3
      REAL=E*A +B+E*A*B+F*D*(3.0*A+1.)
      RIMA= F+G*A**2-F*B*(4.*A+1.)+G*B*A**2
     1 +D+E*D*(A+2.*A**2)
      S=REAL**2 + RIMA**2
      T(I)=1.0/SQRT(S)
      IF ( X(I).GT.10.) GO TO 70
      X(I+1)=X(I)+0.1
100   CONTINUE
      PRINT 90 ,A,C,Y
90    FORMAT (/,10X,F4.2,10X,F5.2,10X,F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.2
      IF (Y.LT.3.) GO TO 33
      Y=0.0
      C=C+0.5
      IF (C.LT.5.) GO TO 33
      C=0.0
      STOP
      END
```

Table 2.1 Computer program of a Mixed Lumped-Distributed Constant K Low-pass Filter

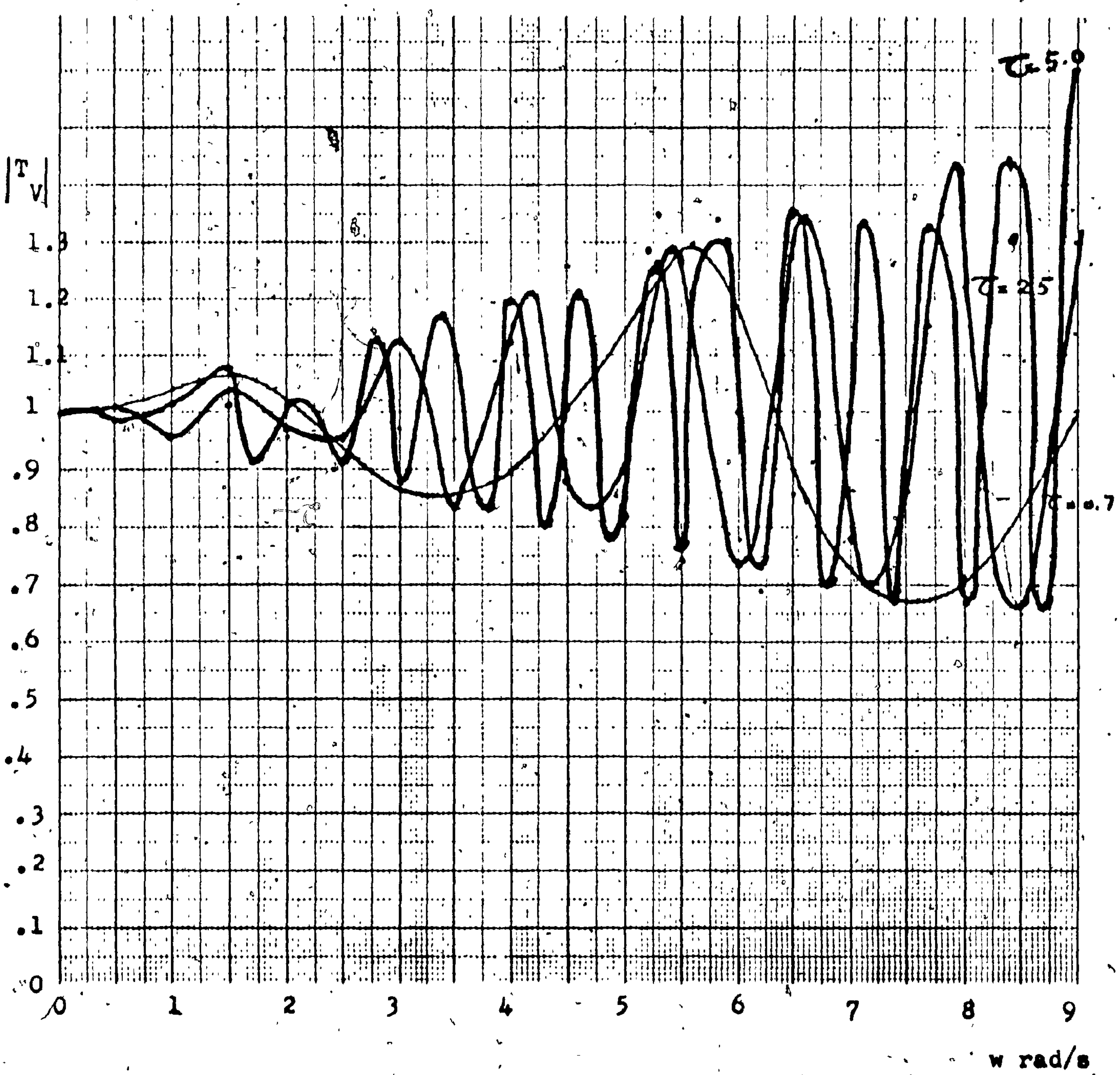

Figure 2.2.1 Response of Constant K Low-Pass Fliter.

K=0.1, C=0.1, and τ=0.7 to 5

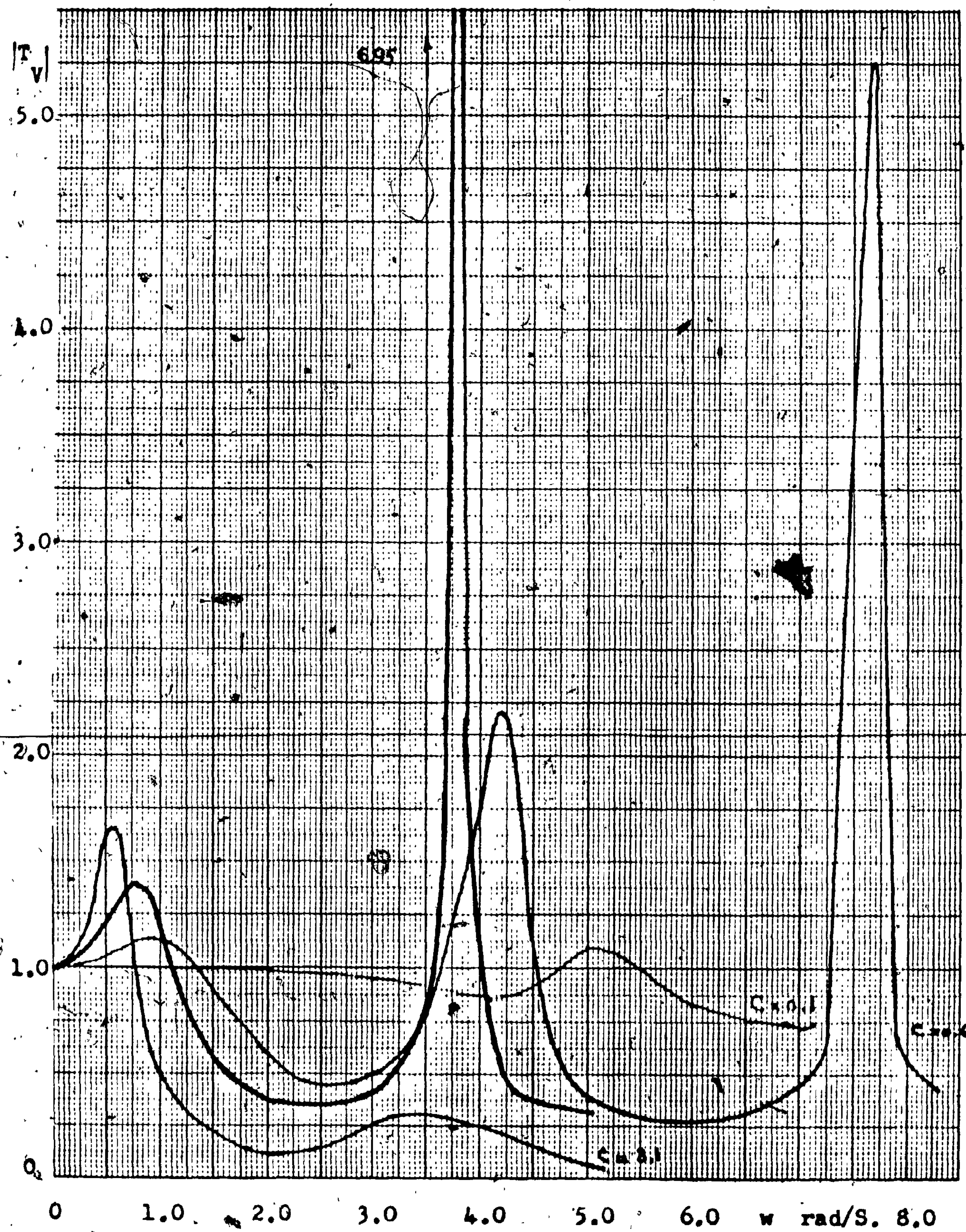

Figure 2.2.2 Response of Constant K Low-Pass Filter.

K-0.1, C-1 and τ-0.8

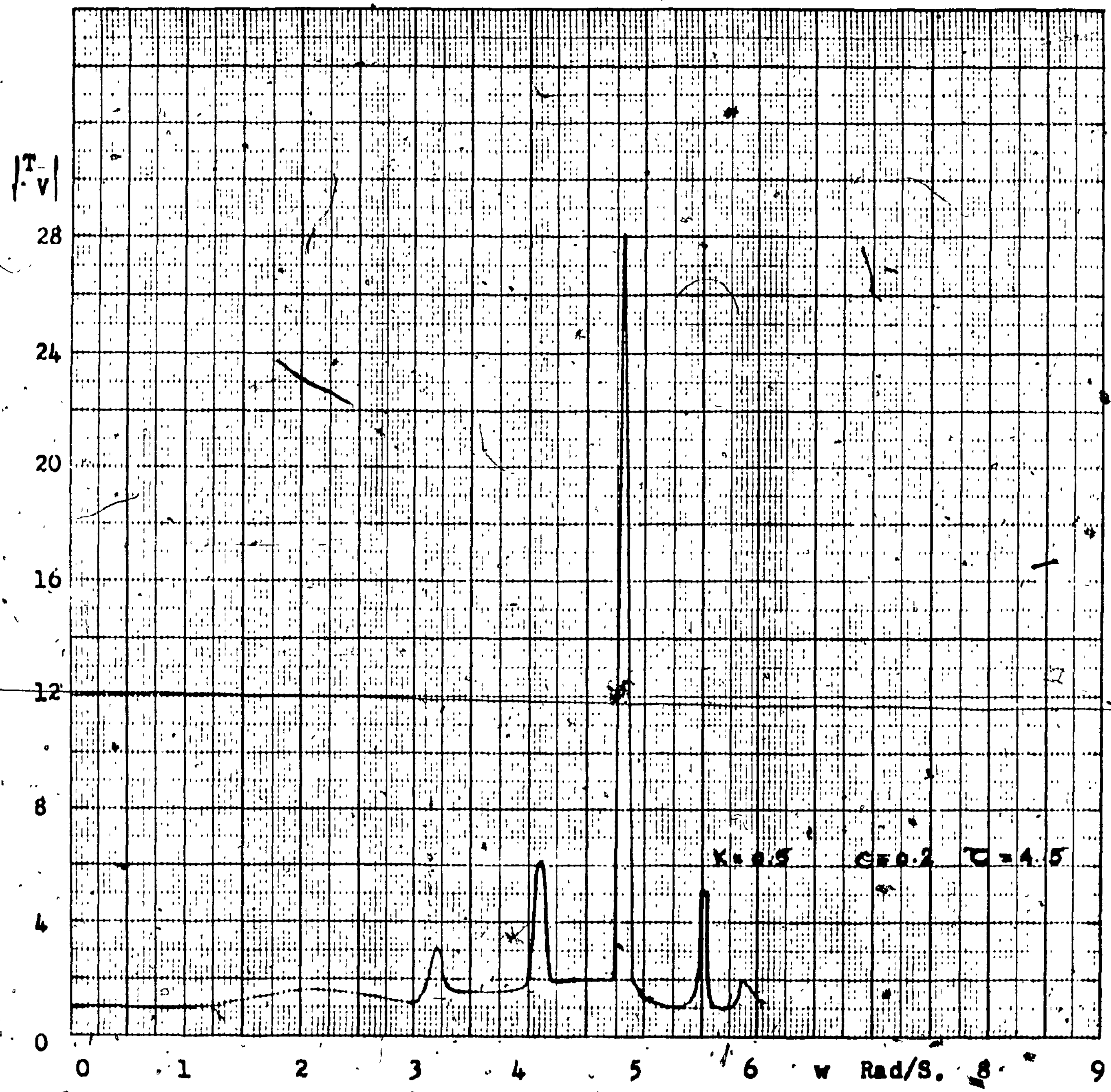

Figure 2.2.3 Response of Constant K Low-Pass Filter.

K=0.5, C=0.2 and τ =4.5

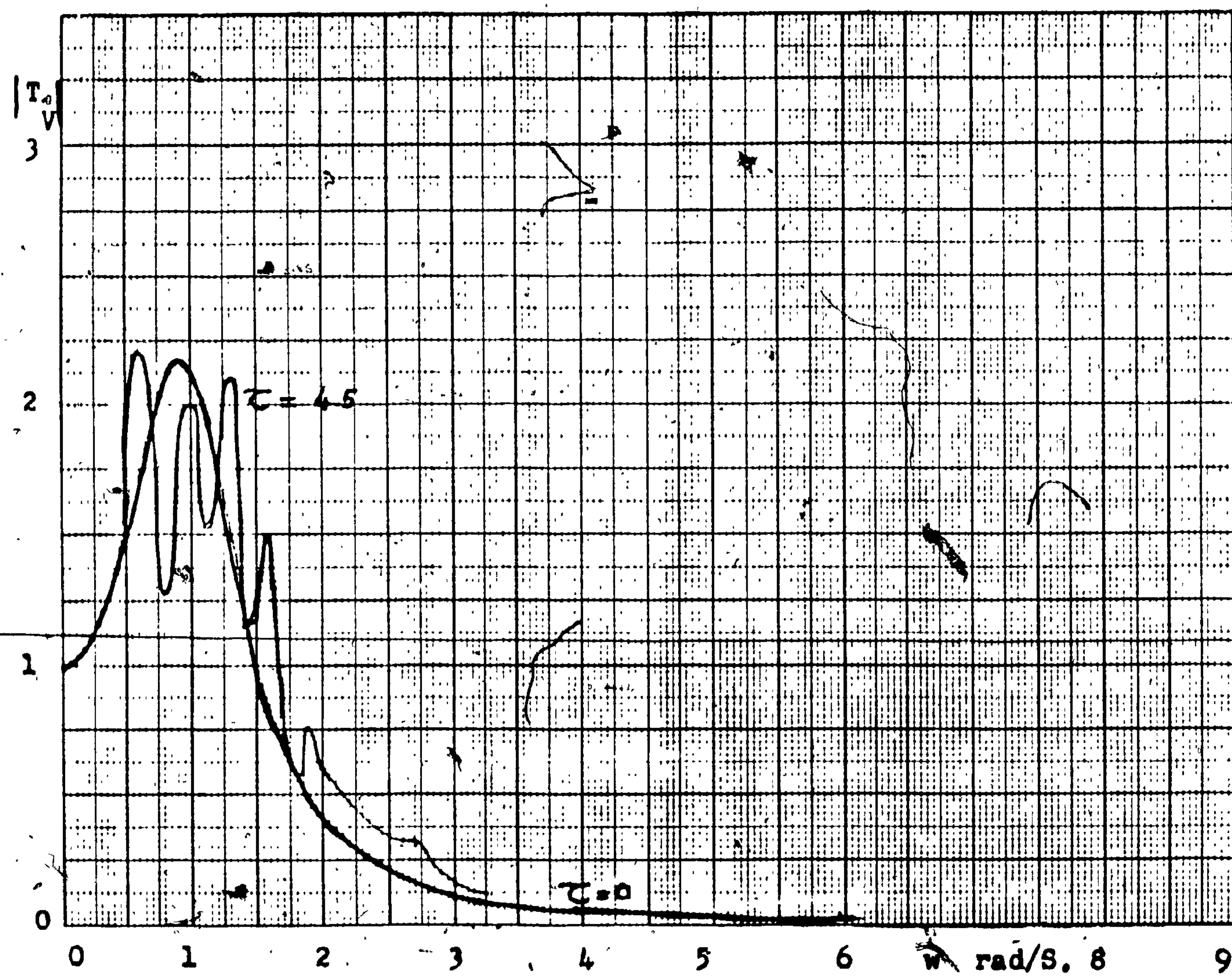

Figure 2.2.4 Response of Constant K Low-Pass Filter.

K=0.5, C=1 and τ =4.5

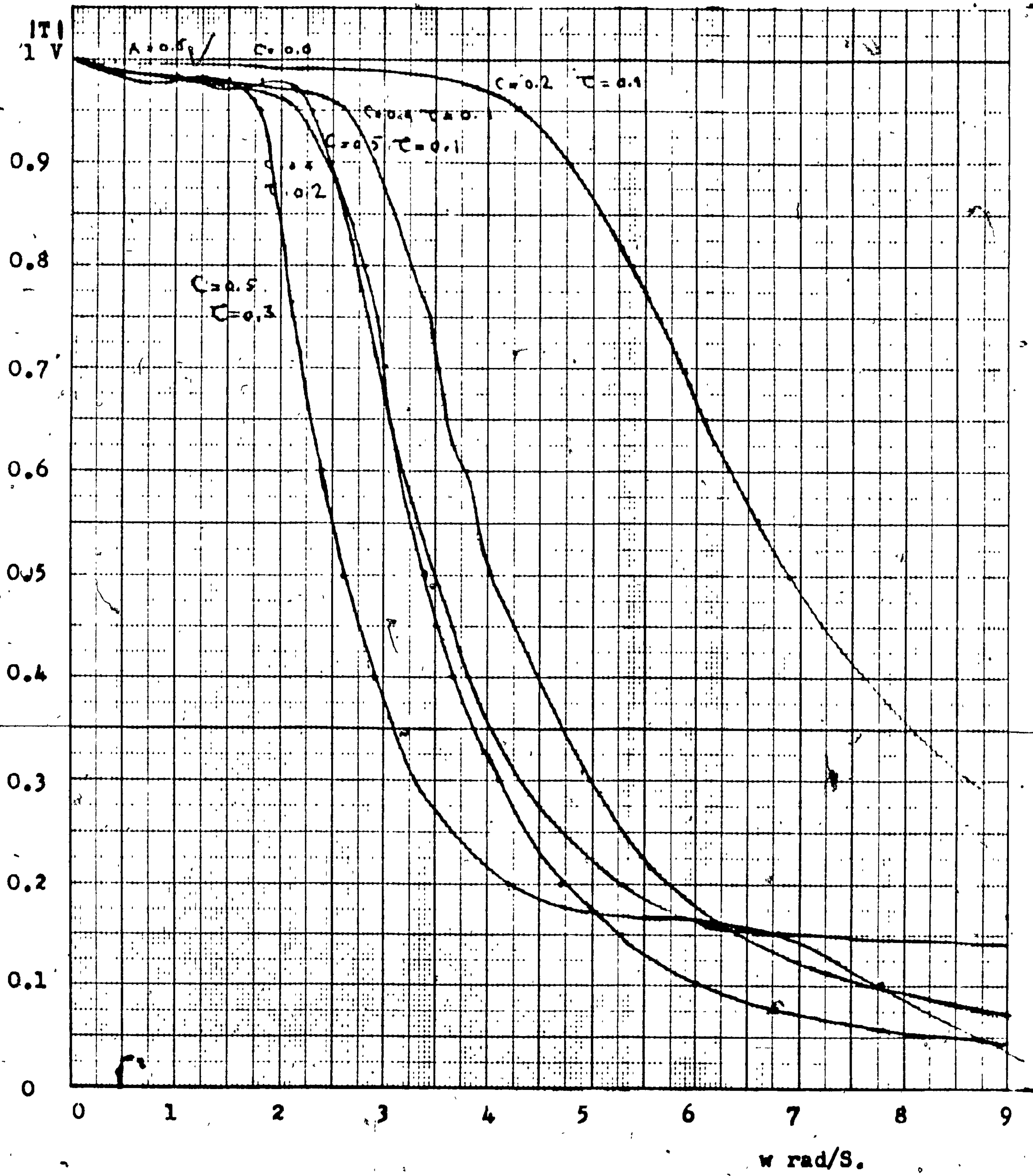

Figure 2.2.5 Response of Constant K Low-Pass Filter.

K=0.6, C=0.2 to 0.5 and τ =0.1 to 0.3

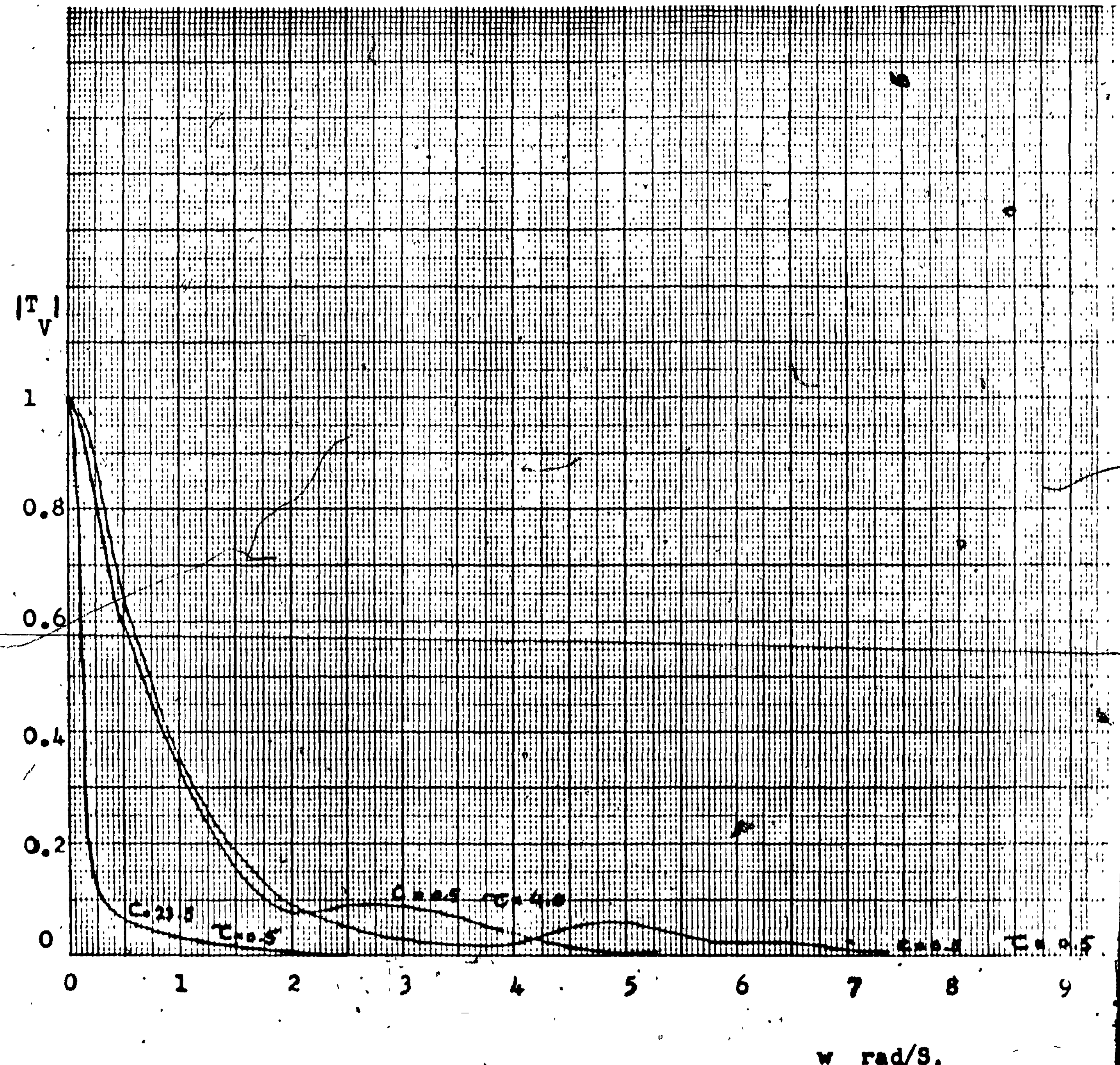

Figure 2.2.6 Response of Constant K Low-Pass Filter.

K=0.5, C=0.5 to 23.5 and τ =0.5 to 4

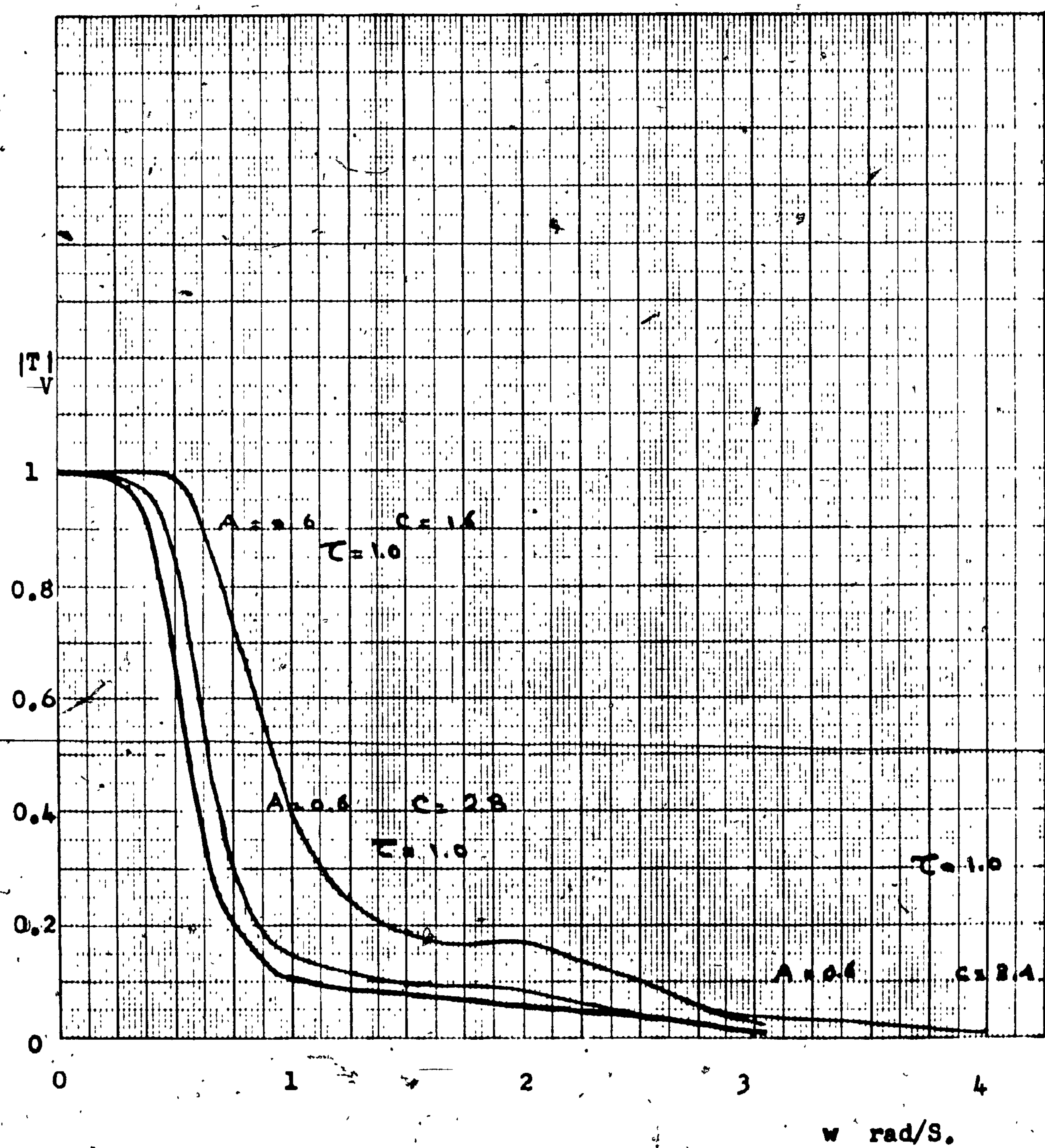

Figure 2.2.7 Response of Consta[illegible] K Low-Pass Filter.

K=0.6, C=1.6 to 3.4 and τ [illegible]

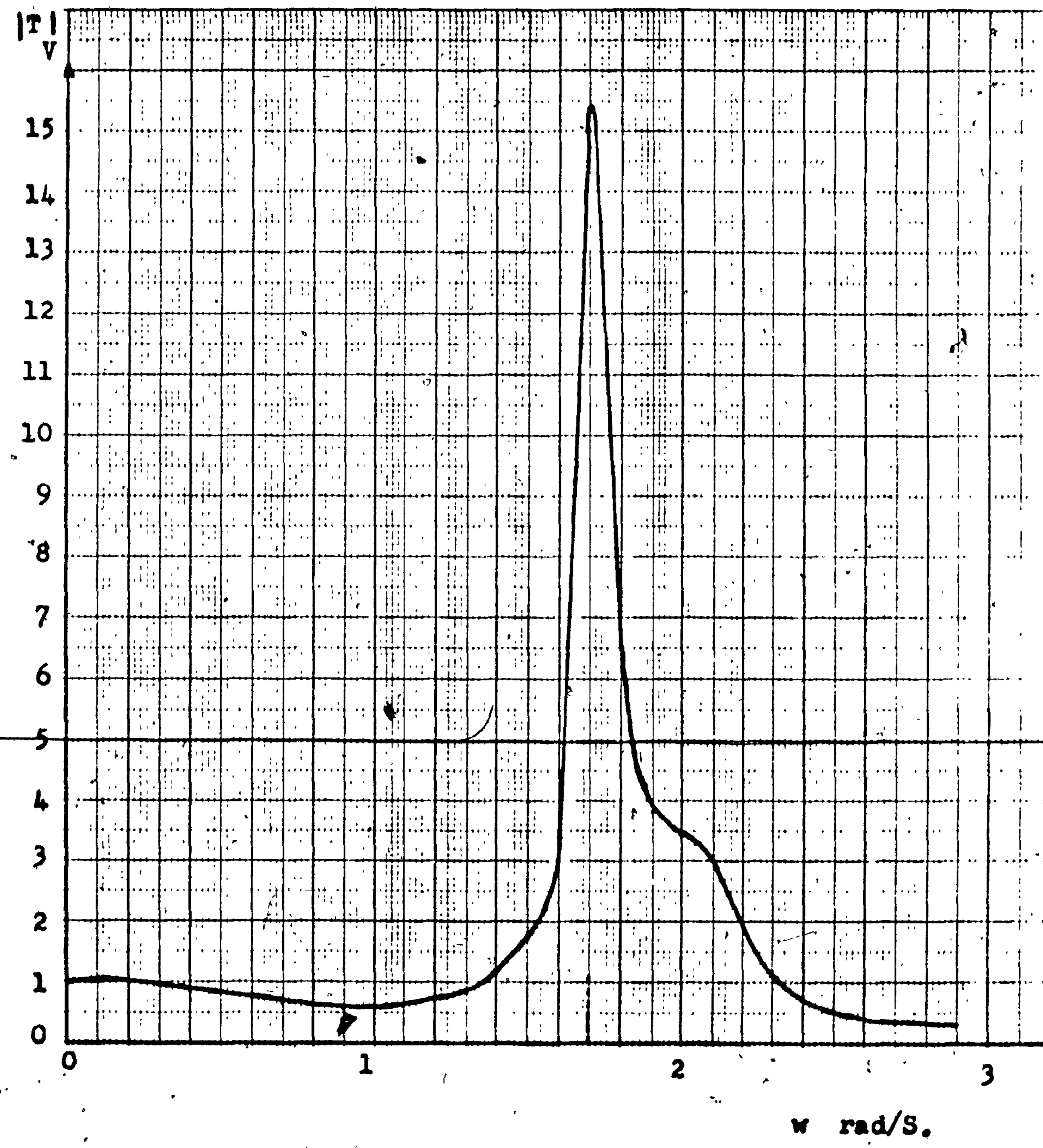

Figure 2.2.8 Response of Low-Pass Constant K Filter.

K= 1.5, C=0.5 and τ=1.2

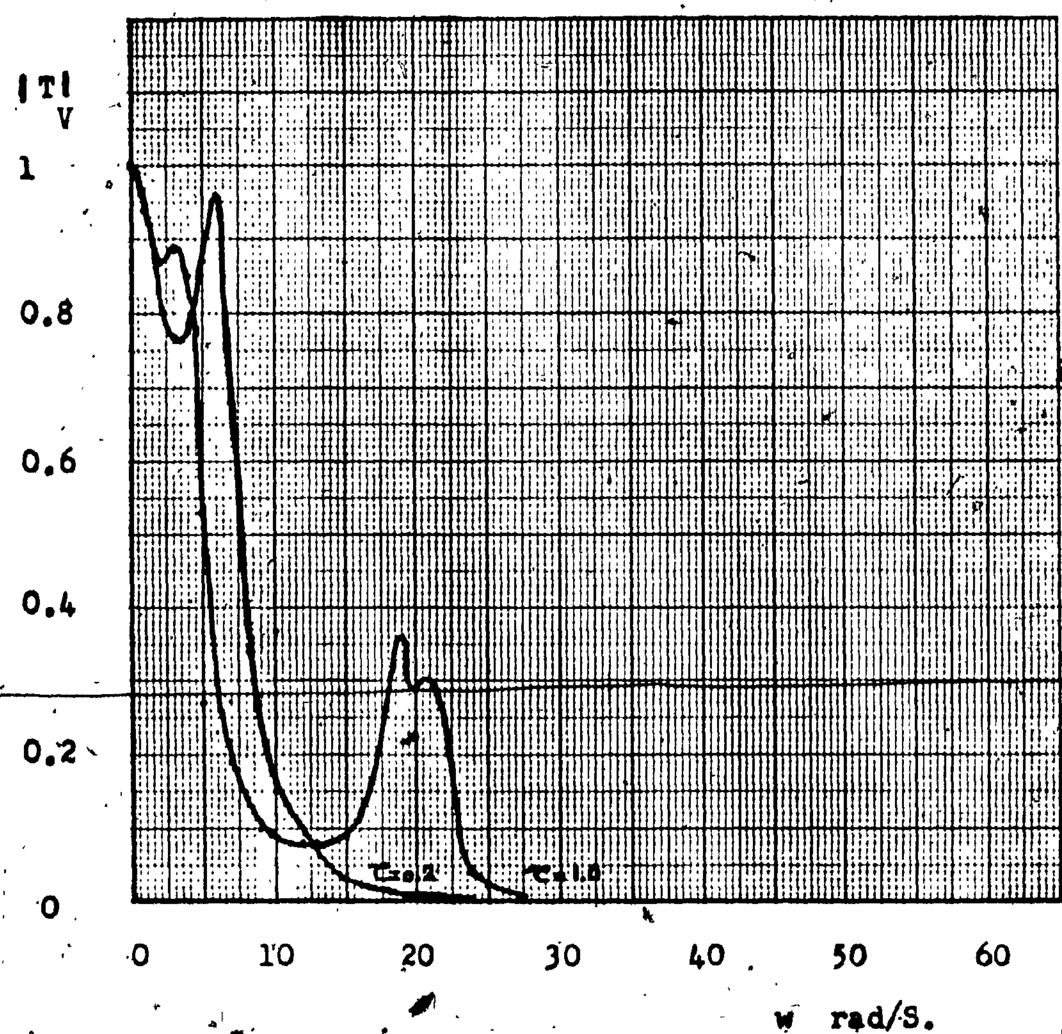

Figure 2.2.9 Response of Low-Pass Constant K Filter.

K=1.5, C=0.2 and τ =0.2 to 1

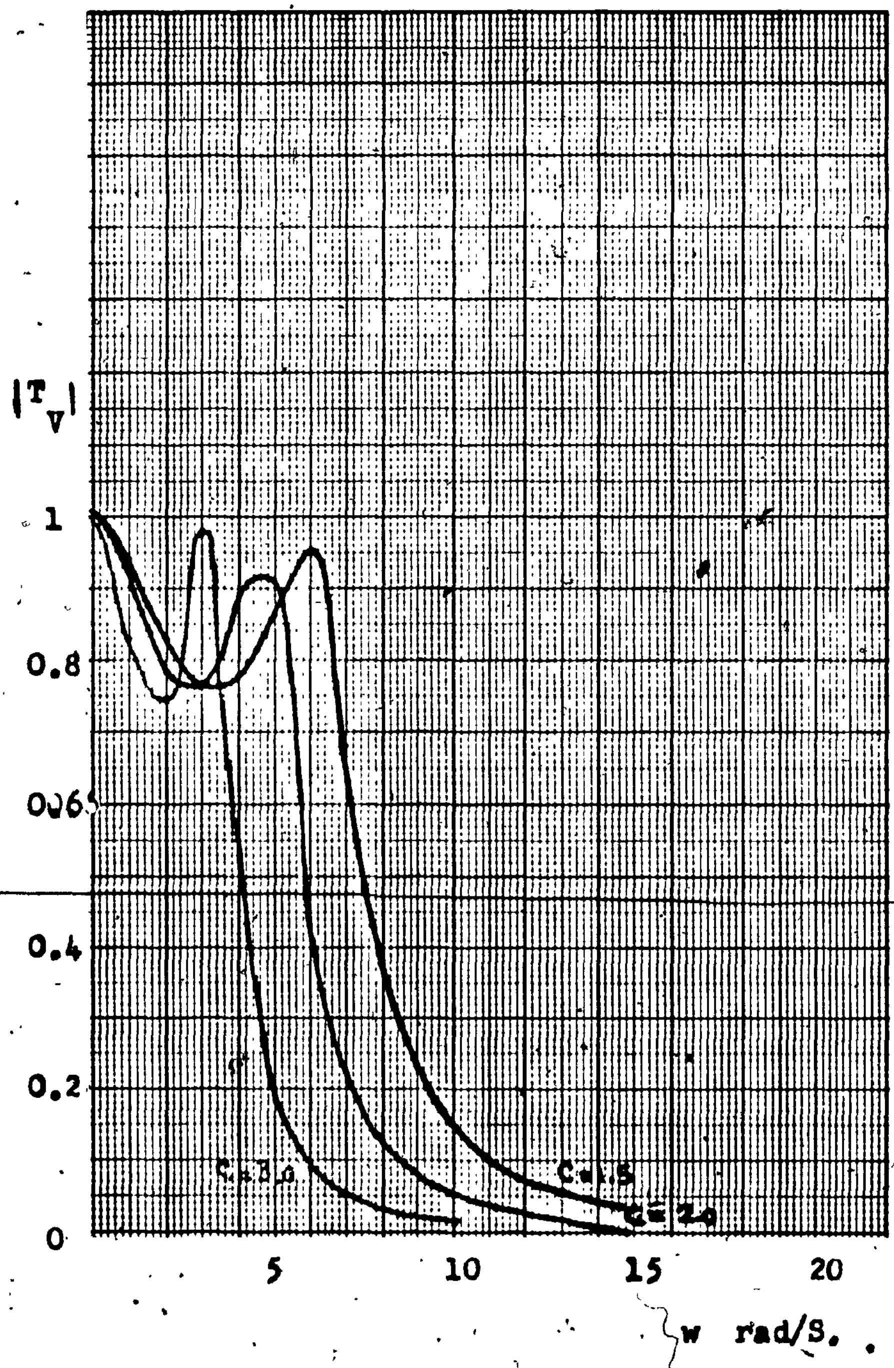

Figure 2.2.10 Response of Low-Pass Constant K Filter.

K=1.5, C=1.5 to 3 and τ =0.2

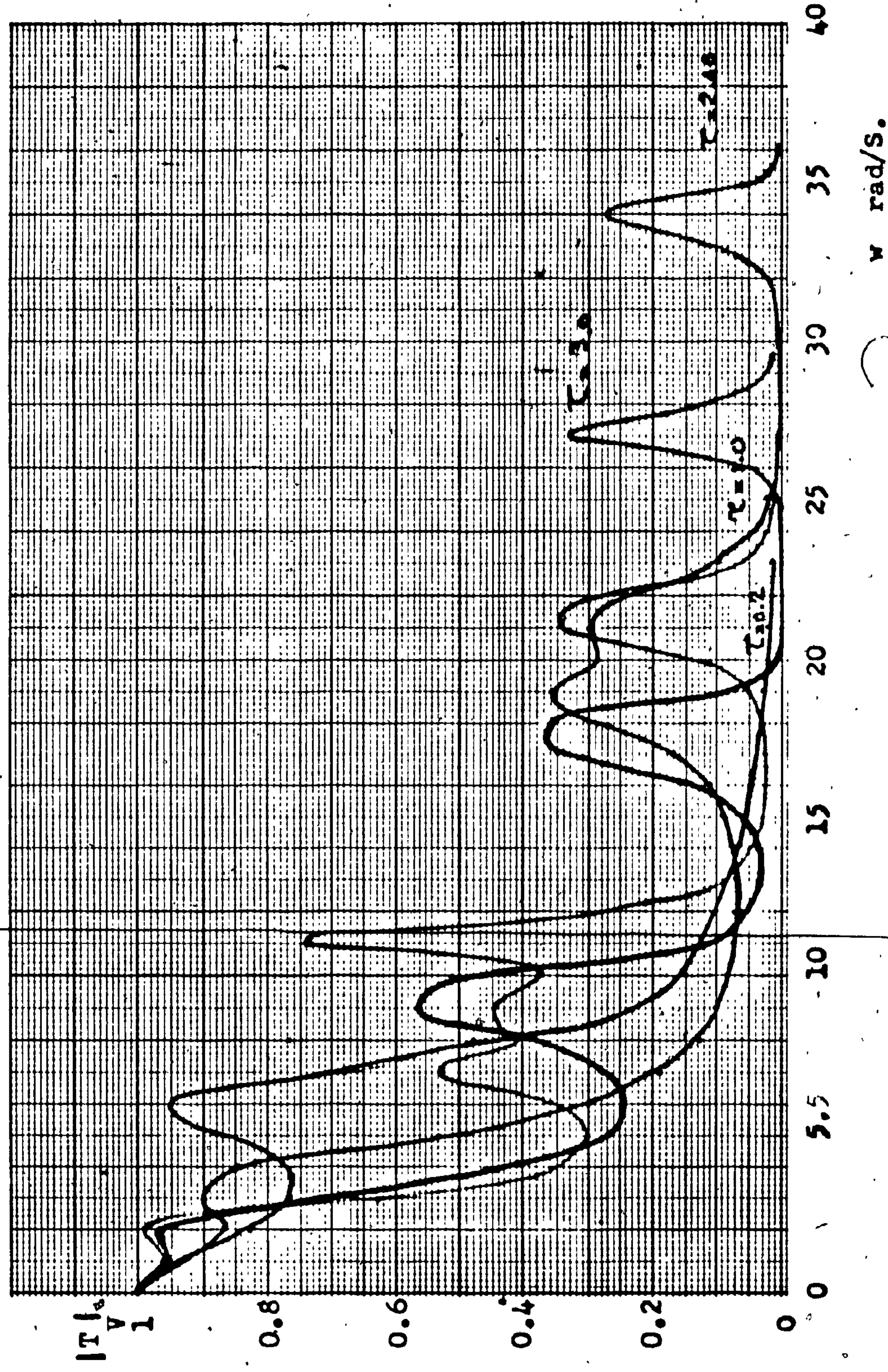

Figure 2.2.11 Response of Low-Pass Constant K Filter.

K=1.5, C=1.5 and τ =0.2 to 3

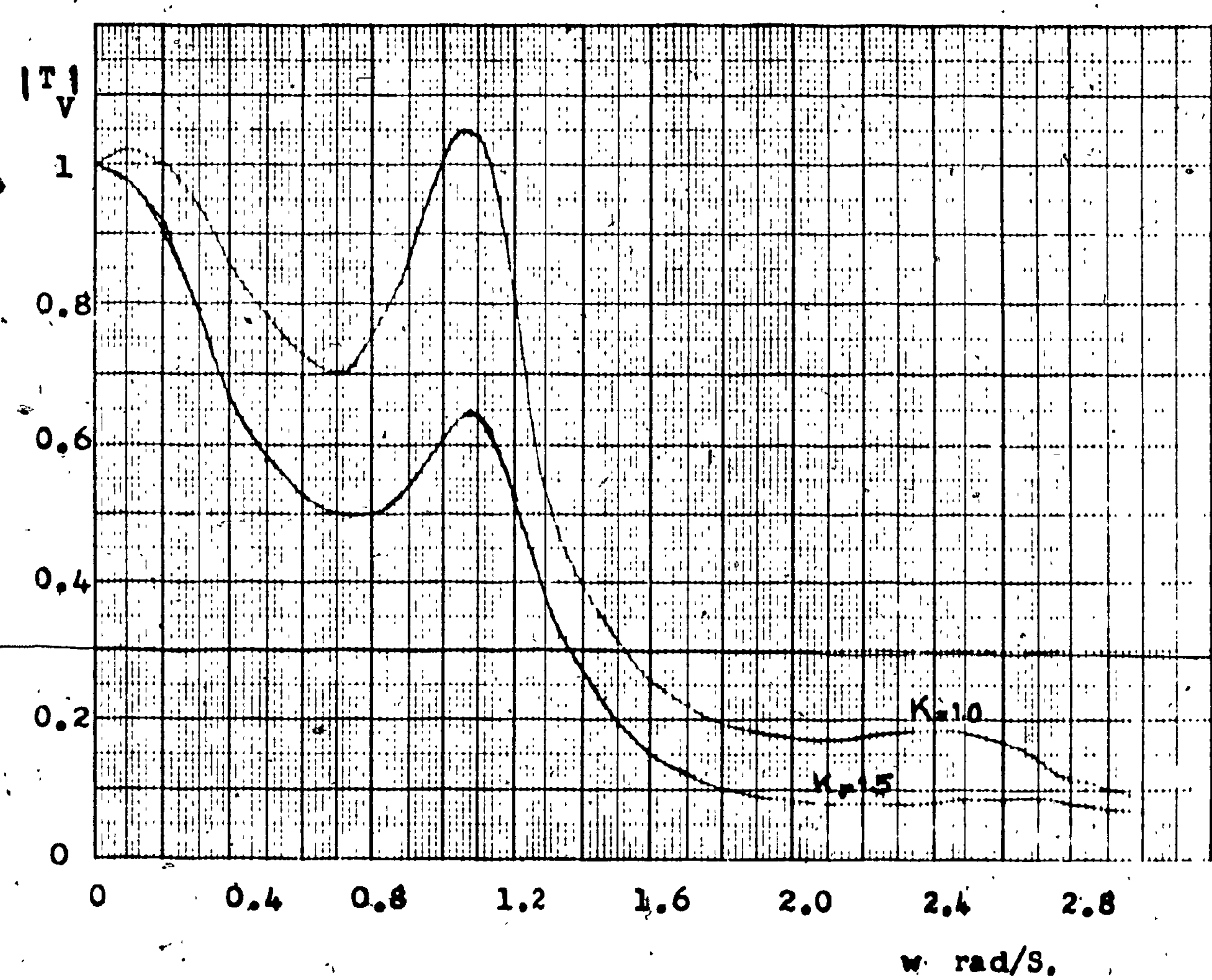

Figure 2.2.12 Response of Low-Pass Constant K Filter.

K=1 to 1.5 , C=1 and τ =1

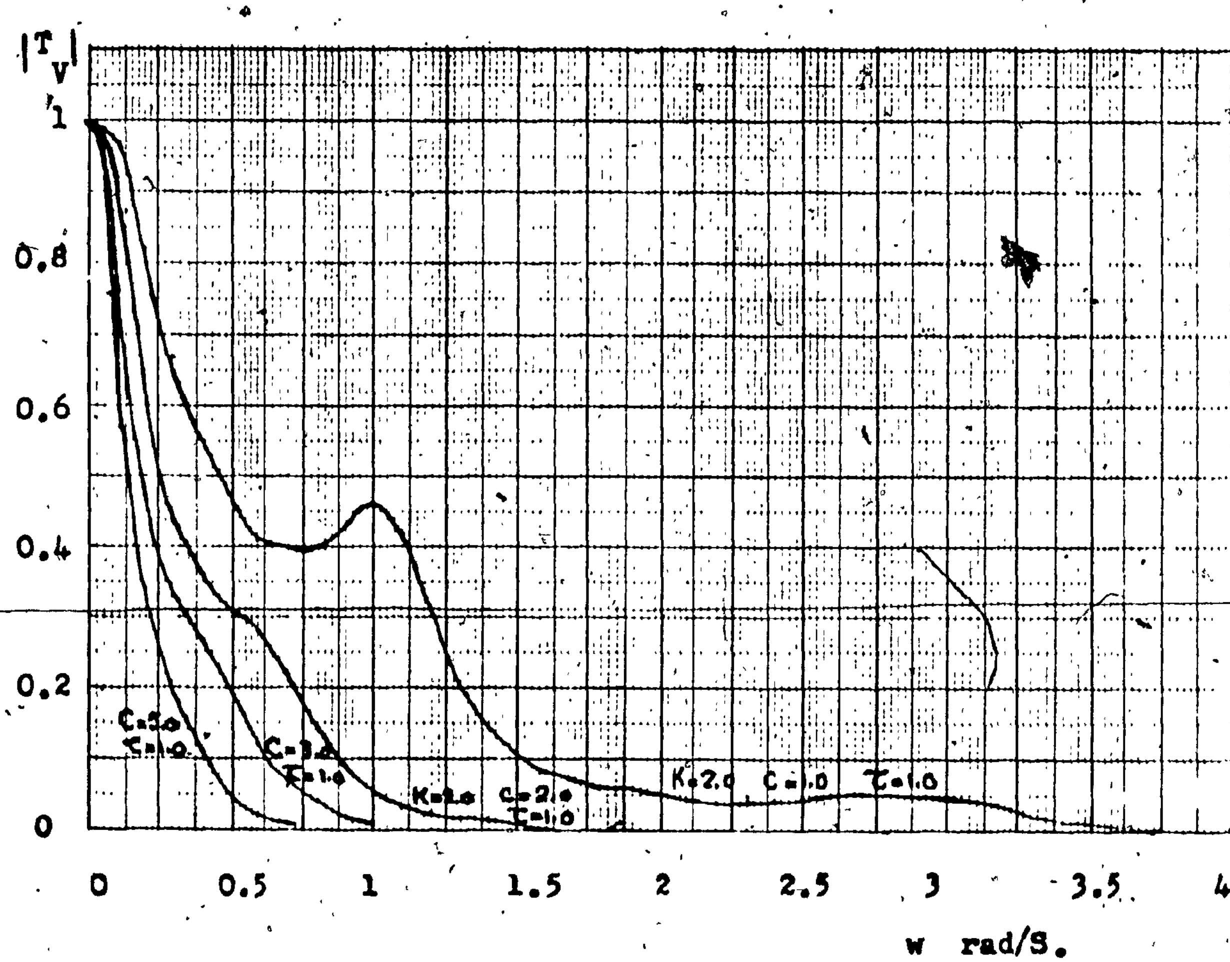

Figure 2.2.13 Response of Low-Pass Constant K Filter.

K=2 , C=1 to 5 and τ =1

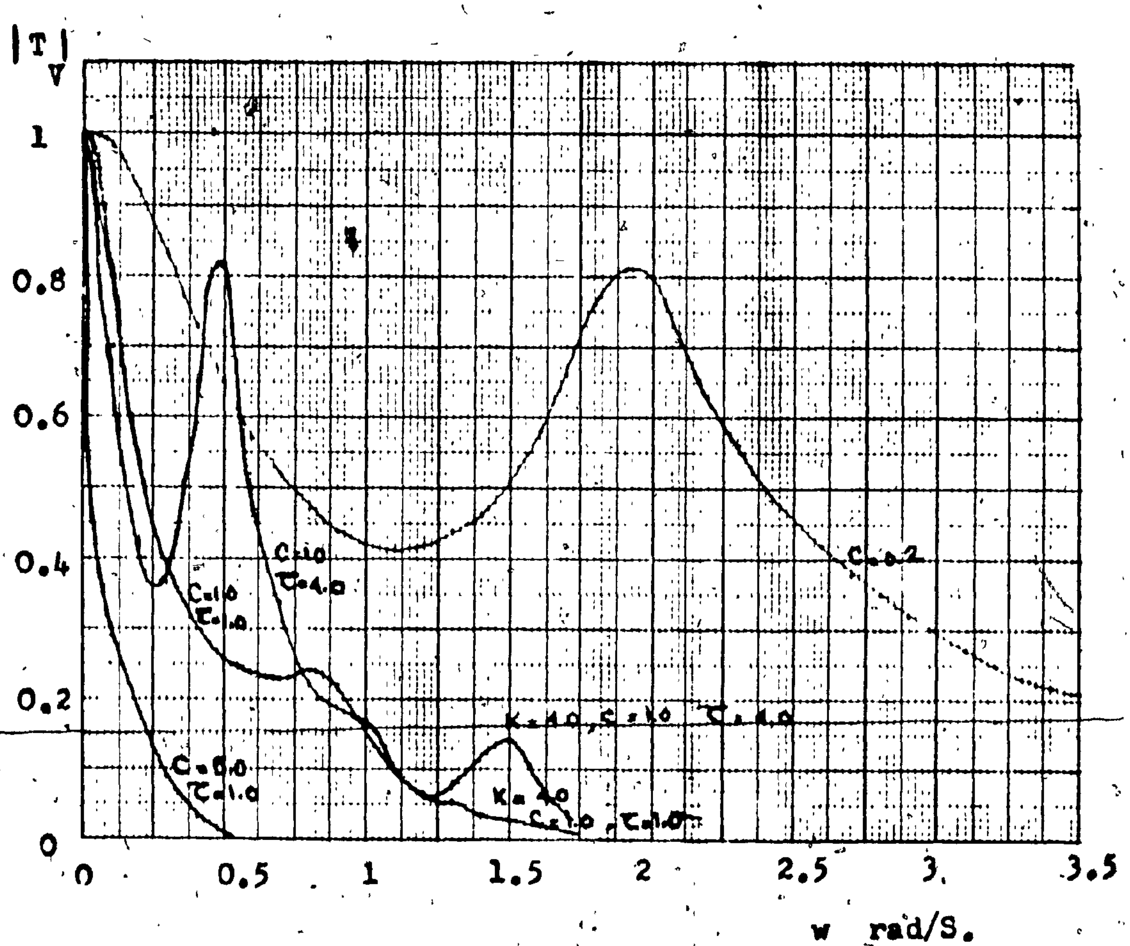

Figure 2.2.14 Response of Low-Pass Constant K Filter.

K=4, C=2 to 5 and τ =1, 4

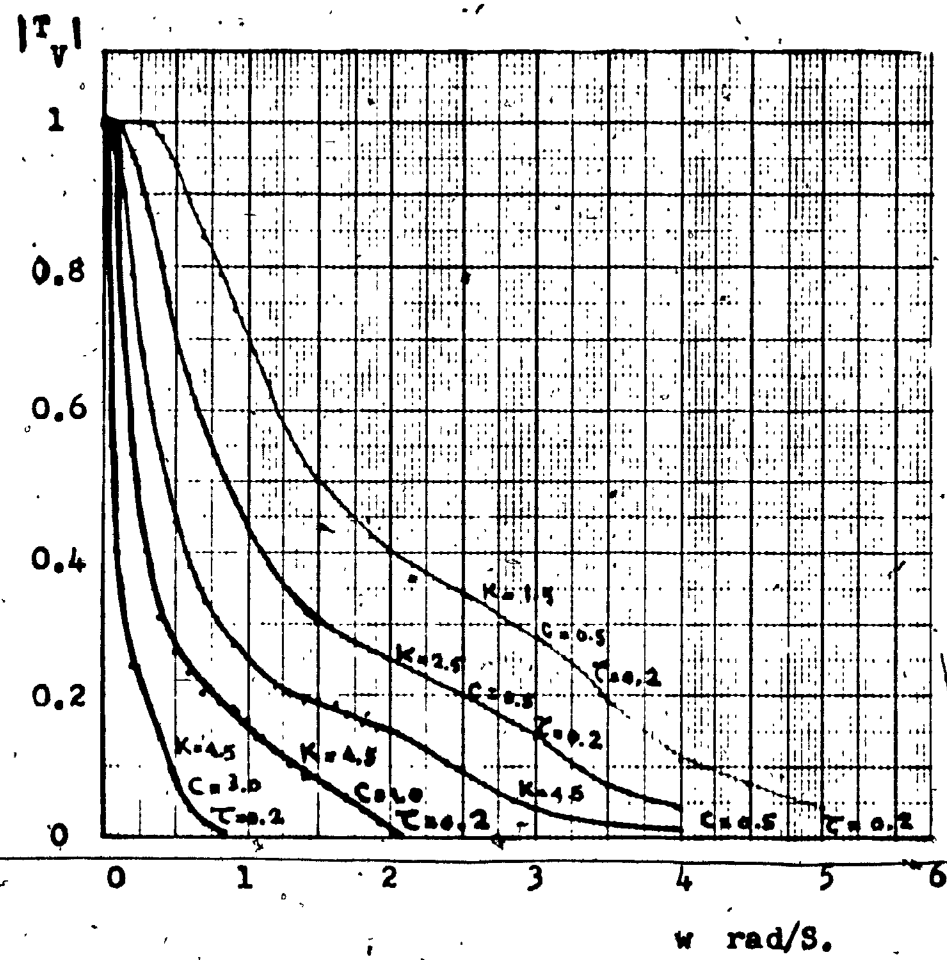

Figure 2.2.15 Response of Low-Pass Constant K Filter.

K=1.5 to 4.5, C=0.5 to 3 and τ=0.2

earlier.

Figure 2.2.15 shows the general tendency of the decrease of the cut-off frequency with the increase of either K or C.

2.2.2 The Butterworth Case

The lumped sections form a Butterworth low-pass filter, for which $L_1 = 0.5$, $L_2 = 0.75$ and $C = 1.5$. Hence the transfer function $|T_V|$ becomes:

$$|T_V| = 1 / \mathrm{SQRT}\left(1 - w\left(\frac{5}{4}\mathrm{Sin}(2w\tau) - \frac{19}{4}\mathrm{Cos}(2w\tau)\right)\right.$$
$$- w^2 \frac{3}{16}\left(1 + \mathrm{Cos}(2w\tau) - 3\,\mathrm{Sin}(2w\tau)\right)$$
$$\left. - w^3 \frac{3}{32}\left(1 - \mathrm{Cos}(2w\tau)\right)\right) \qquad (2.23)$$

The computer program for this case is given in Table 2.2 .

Figure 2.2.16 shows the transfer function response for different τ's. As τ increases the number of oscillations increases, in addition to the frequency of oscillations.

2.2.3 The Lumped Elements are Independent

For L_1, L_2 and C independently set, the transfer function becomes:

$$|T_V| = 1 / \mathrm{SQRT}\left(1 - w\left(C\,\mathrm{Sin}(2w\tau) - \frac{1}{2}L_2\,\mathrm{Sin}(4w\tau)\right)\right.$$
$$+ w^2\left(\left(L_1^2 + \frac{5}{8}L_2^2 + \frac{1}{8}C^2 + \frac{1}{2}L_1L_2 + \frac{3}{4}L_1C\right)\right.$$
$$-\left(2L_1C + L_2C + \frac{1}{8}C^2\right)\mathrm{Cos}(2w\tau)$$

$$
\begin{aligned}
&+\left(\frac{3}{8}L_1 C+\frac{1}{4}L_2 C+\frac{3}{2}L_1 L_2\right)\text{Cos}\,(4w\tau)\,)\\
&+w^3\left(\left(\frac{3}{2}L_1^2 C+\frac{1}{2}L_1 L_2 C+\frac{1}{2}L_2^2 C\right)\text{Sin}\,(2w\tau)\right.\\
&-\left(\frac{5}{4}L_1 L_2 C+\frac{1}{2}L_1 L_2^2+\frac{1}{2}L_1 L_2^2\right)\text{Sin}\,(4w\tau)\,)\\
&+w^4\left(\left(\frac{1}{2}L_1^2 L_2^2+\frac{3}{4}L_1^2 C^2-\frac{1}{2}L_1 L_2^2 C-\frac{3}{4}L_1 L_2 C^2\right)\right.\\
&-\left(\frac{1}{2}L_1^2 L_2^2+\frac{1}{4}L_1 L_2 C^2+\frac{1}{2}L_1 L_2^2 C+L_1^2 L_2 C\right)\text{Cos}\,(4w\tau)\,)\\
&+w^5\left(\left(\frac{1}{2}L_1^2 L_2 C^2+L_1^2 L_2^2 C\right)\text{Sin}\,(2w\tau)\right)\\
&+w^6\left(\frac{5}{8}+\frac{1}{2}\text{Cos}\,(2w\tau)+\frac{1}{8}\text{Cos}\,(4w\tau)\right)\left(L_1^2 L_2^2 C^2\right)
\end{aligned}
\tag{2.24}
$$

The computer program is given in Table 2.3 .

Figure 2.2.17 shows the response for $L_1=0.5$, $L_2=0.1$ and $C=0.1$ and $\tau=0.2$ to 1.6. It is noticed that the response is smoother with $\tau=0.2$, the effect of increasing τ is a decrease in the cut-off frequency and the appearance of oscillations.

Figure 2.2.18 shows the effect of increasing L_2 and C while L_1 and τ are kept constant. ($L_1=0.5$, $L_2=1.5$, $C=1.25$ and $\tau=0.4$). The oscillations increase.

Figure 2.2.19 gives the effect of increasing C condiderably ($C=21.5$). A very smooth response is obtained with a fast cut-off.

Figure 2.2.20 gives a family of curves for $L_1=0.5$, $L_2=0.5$, $\tau=0.5$ and C ranging from 21 to 49.5 . We deduce that we can design a low-

```
      PROGRAM FAROUK (INPUT,OUTPUT)
C     STUDY OF A L.P.F. OF MIXED LUMPED DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=0.5
      B=1.5
      C=0.75
      Y=0.0
33    X(1)=0.1
      DO 100 I=1,100
      REA=-0.5*X(I)**2*A*C
     1 +0.5*(2.-X(I)**2*A*C)   *COS(2.*X(I)*Y)
     1 -0.5*SIN(2.*X(I)*Y)*X(I)*(2.*A+B+C)
      RIMA=-0.5*(X(I)*C-X(I)**3*A*B*C)
     1 +0.5*COS(2.*X(I)*Y)*(X(I)*(2.*A+2.*B+C)-X(I)**3*A*B*C)
     1 +0.5*SIN(2.*X(I)*Y)*(2.-X(I)**2*(A*C+2.*A*B))
      S=REA**2+RIMA**2
      T(I)=1.0/SQRT(S)
      IF (T(I).GT.1.) GOTO 70
      IF ( X(I).GT.10.) GO TO 70
      X(I+1)=X(I)+0.1
100   CONTINUE
      PRINT 90,A,B,C,Y
90    FORMAT(/,10X,F4.2,10X,F4.2,10X,F4.2,10X,F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.2
      IF (Y.LT.3.) GO TO 33
      Y=0.0
      C=C+0.5
      IF (C.LT.5.) GO TO 33
      C=0.0
      STOP
      END
```

Table 2.2 Computer program of a Mixed Lumped-Distributed Butterworth Low-pass Filter

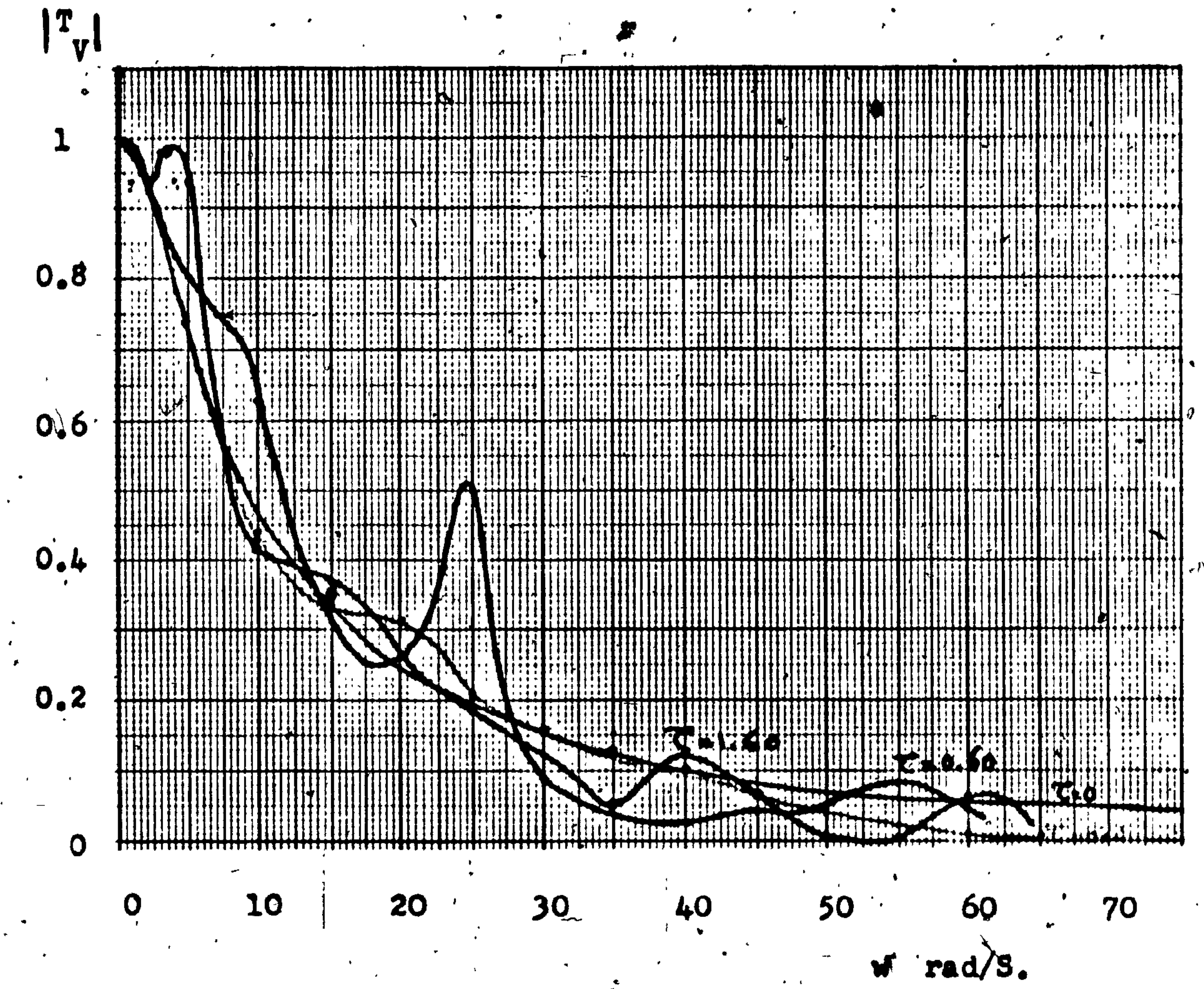

Figure 2.2.16 The lumped section forms a Butterworth low-pass filter. L_1=0.5, L_2=0.75 and C=1.5. τ = 0.2 to 1.6 .

pass filter having a smooth response with fast cut-off if τ is kept low, below 0.5, and C is high, over 21.

Figure2.2.21 gives the response for higher L_1 and L_2 (L_1=5, L_2=0.5), with increased C and τ. We note that a low-pass filter is obtained with any value of L_2, C and τ because of the high inductive reactance at the input of the filter.

Figure 2.2.22 gives the response for L_1=1, L_2=0.1, C=0.1 and τ =0.2 to 3 . As τ is increased, the number of oscillations increase due to mismatch.

Figure 2.2.23 gives the response for L_1=0.5, L_2=0.1, C=0.1 and τ =0.2 to 1.6 . As τ increases, the number of oscillations increase.

Figure 2.2.24 gives the response for L_1=0.5, L_2=1.5, C=0.1 and τ =0.2 to 1.6 . At τ =0.2, oscillations are less important than at τ =1.6, however at τ =0.6 a high oscillation occur around w=30 .

Figure 2.2.25 shows the effect of increased C (C=1.5 to 2.5). An acceptable low-pass filter is obtained.

2.3 Discussions

In this chapter, we have considered a particular type of mixed lumped-distributed low-pass filters. It is found that only certain values of elemental values and time delays give rise to low-pass filters.

```
      PROGRAM FAROUK (INPUT,OUTPUT)
C     STUDY OF A L.P.F. OF MIXED LUMPED DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=1.0
      B=1.0
      C=0.1
      Y=0.0
33    X(1)=0.1
      DO 100 I=1,100
      REA=-0.5*X(I)**2*A*C
     1 +0.5*(2.-X(I)**2*A*C)  *COS(2.*X(I)*Y)
     1 -0.5*SIN(2.*X(I)*Y)*X(I)*(2.*A+B+C)
      RIMA=-0.5*(X(I)*C-X(I)**3*A*B*C)
     1 +0.5*COS(2.*X(I)*Y)*(X(I)*(2.*A+2.*B+C)-X(I)**3*A*B*C)
     1 +0.5*SIN(2.*X(I)*Y)*(2.-X(I)**2*(A*C+2.*A*B))
      S=REA**2+RIMA**2
      T(I)=1.0/SQRT(S)
      IF (T(I).GT.1.) GOTO 70
      IF ( X(I).GT.10.) GO TO 70
      X(I+1)=X(I)+0.1
100   CONTINUE
      PRINT 90,A,B,C,Y
90    FORMAT(/,10X,F4.2,10X,F4.2,10X,F4.2,10X,F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.2
      IF (Y.LT.3.) GO TO 33
      Y=0.0
      C=C+0.5
      IF (C.LT.5.) GO TO 33
      C=0.0
      STOP
      END
```

Table 2.3 Computer program of a Mixed Lumped-Distributed Independent Elements Low-Pass Filter

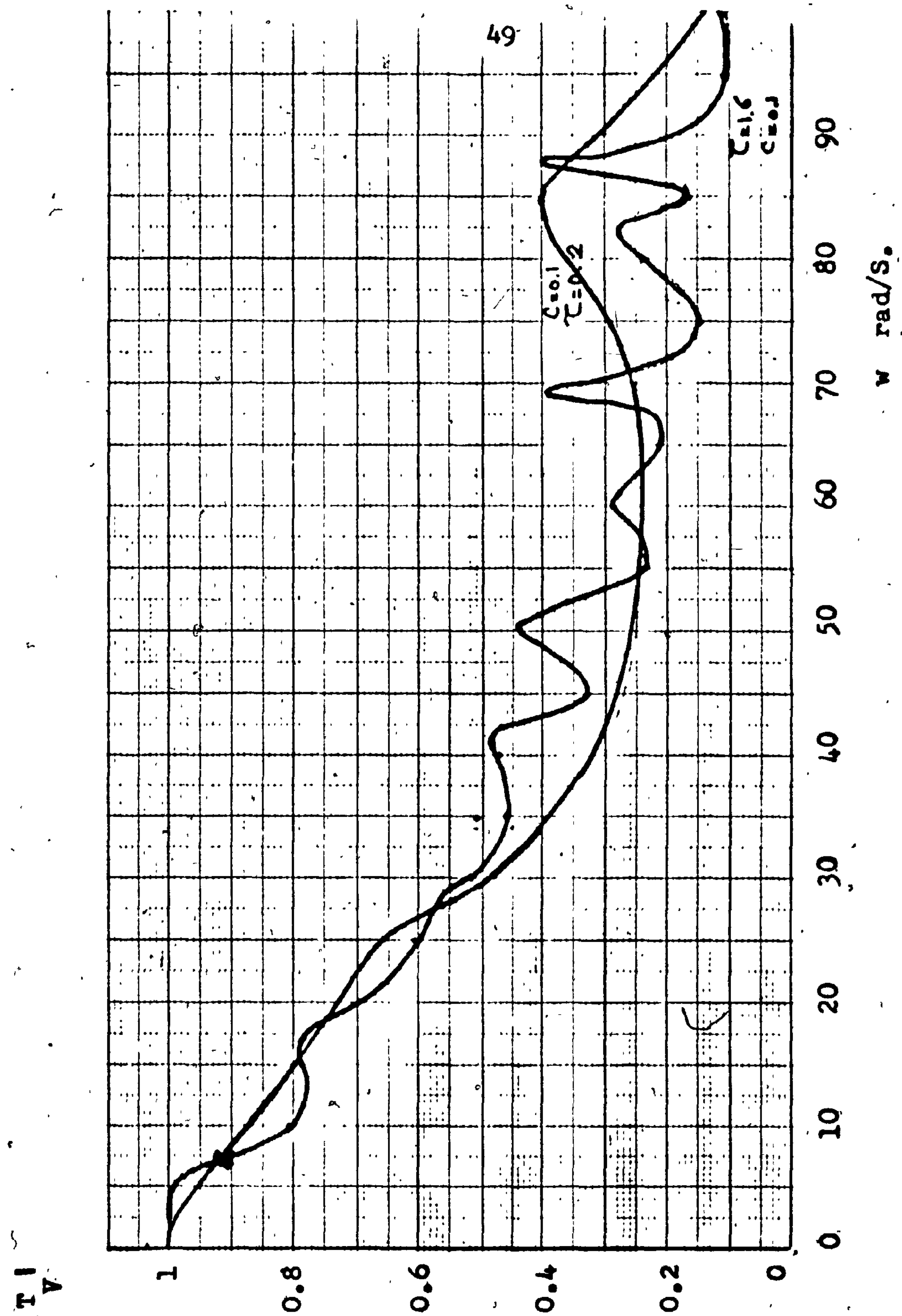

Figure 2.2.17 Response of the low-pass filter where L_1=0.5, L_2=0.1, C=0.1 and τ=0.2 to 1.6

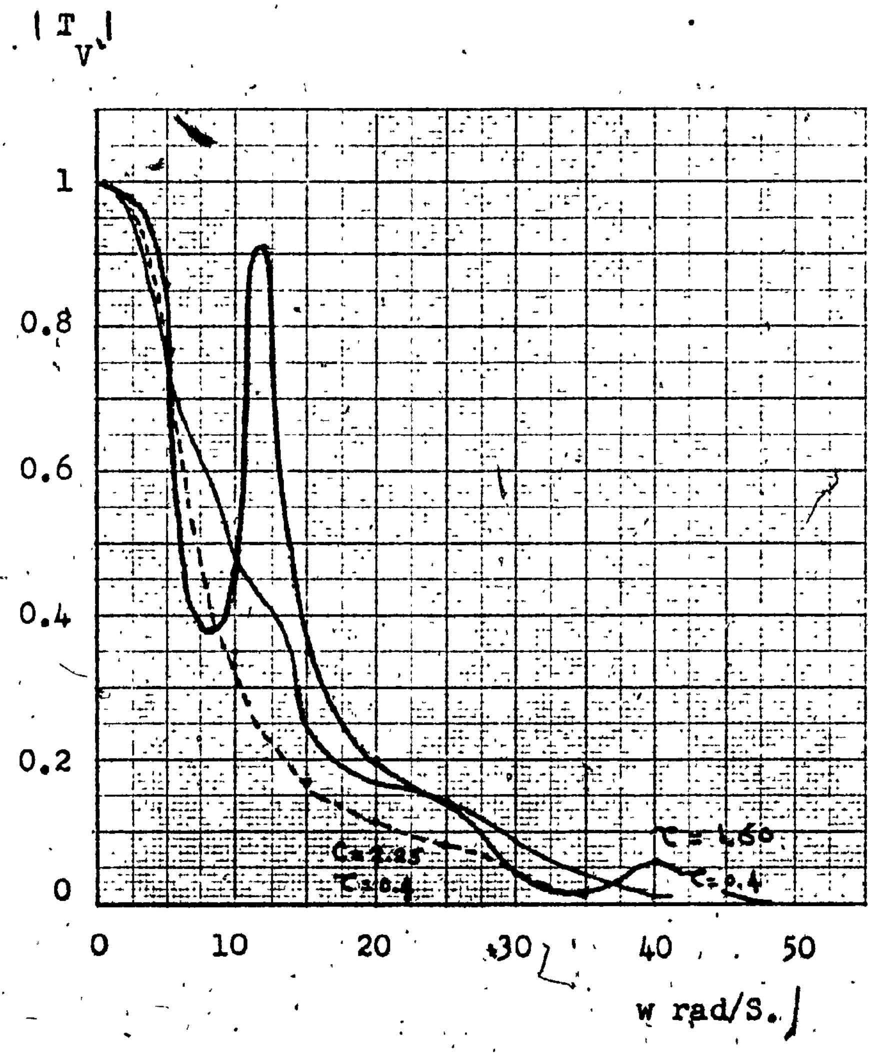

Figure 2.2.18 Response of the low-pass filter for L_1=0.5, L_2=1.5, C=1.25 and τ =0.4 .

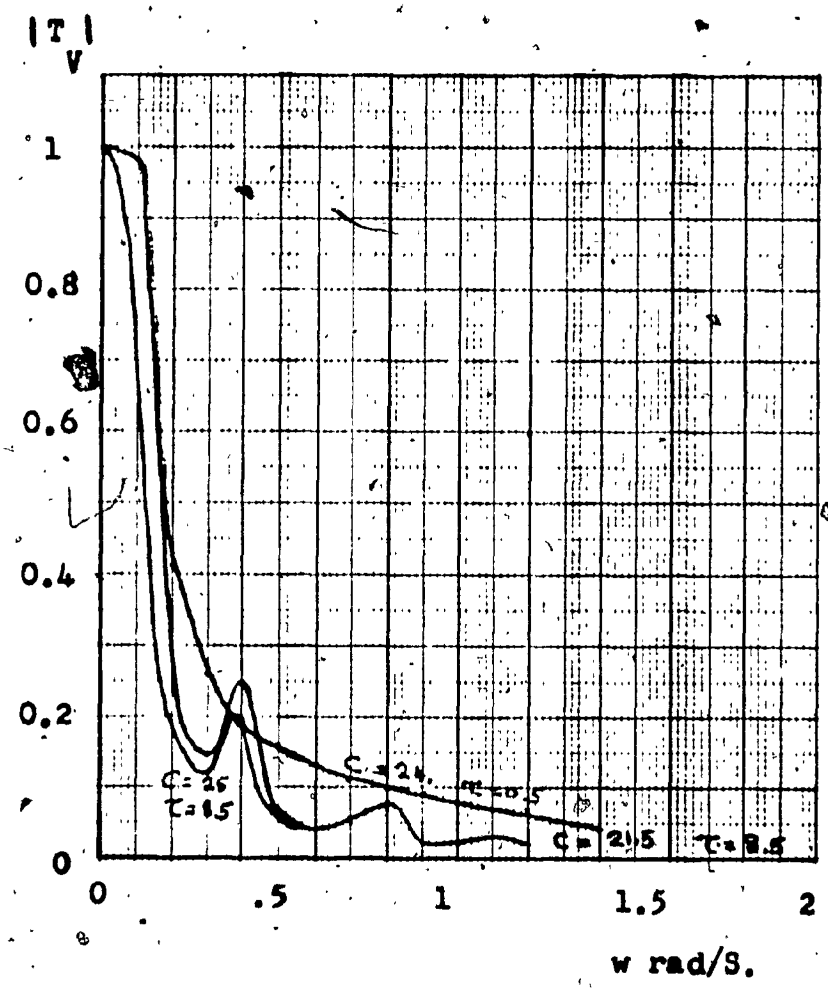

Figure 2.2.19 Response of the low-pass filter for L_1=0.5, L_2=1.5 , C=21.5 and τ=0.5 to 8.5 .

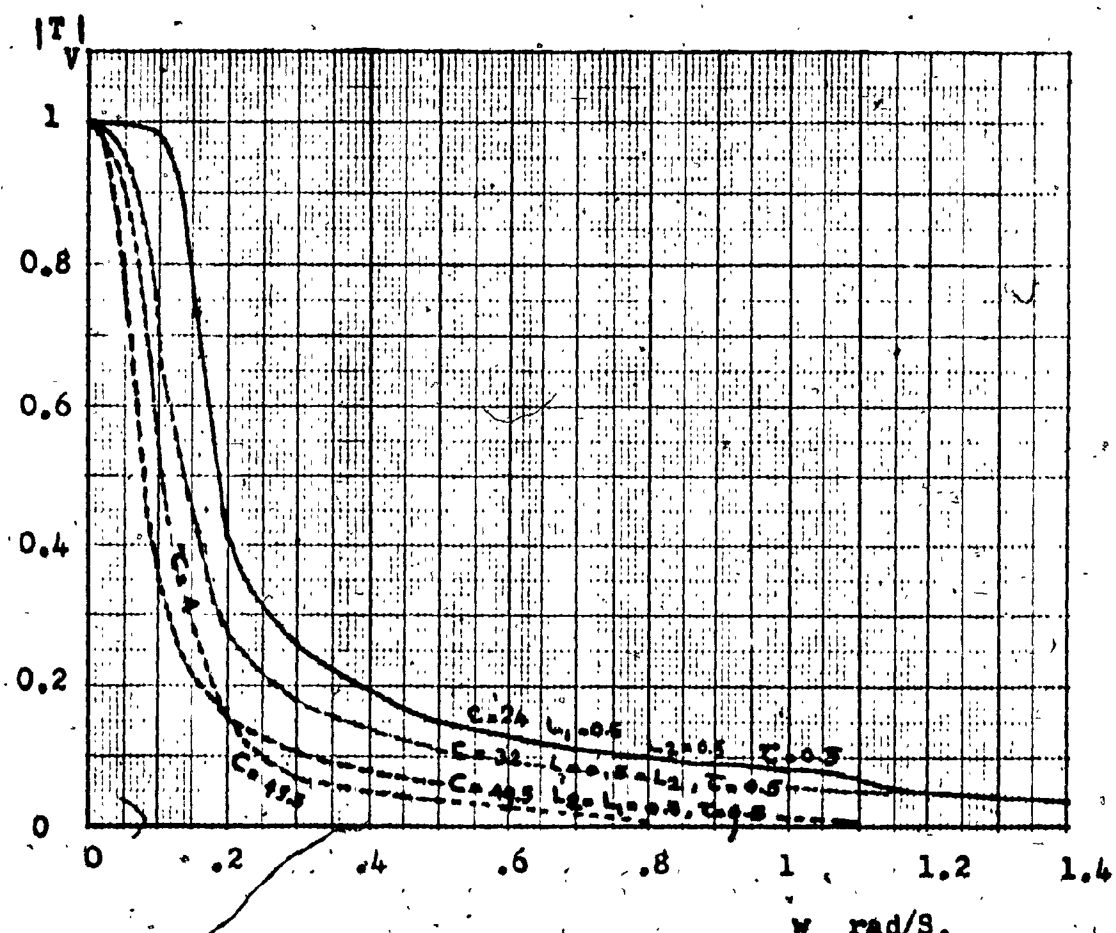

Figure 2.2.20 Response of the low-pass filter for L_1=0.5, L_2=0.5. C=21 to 49.5 and τ =0.5 .

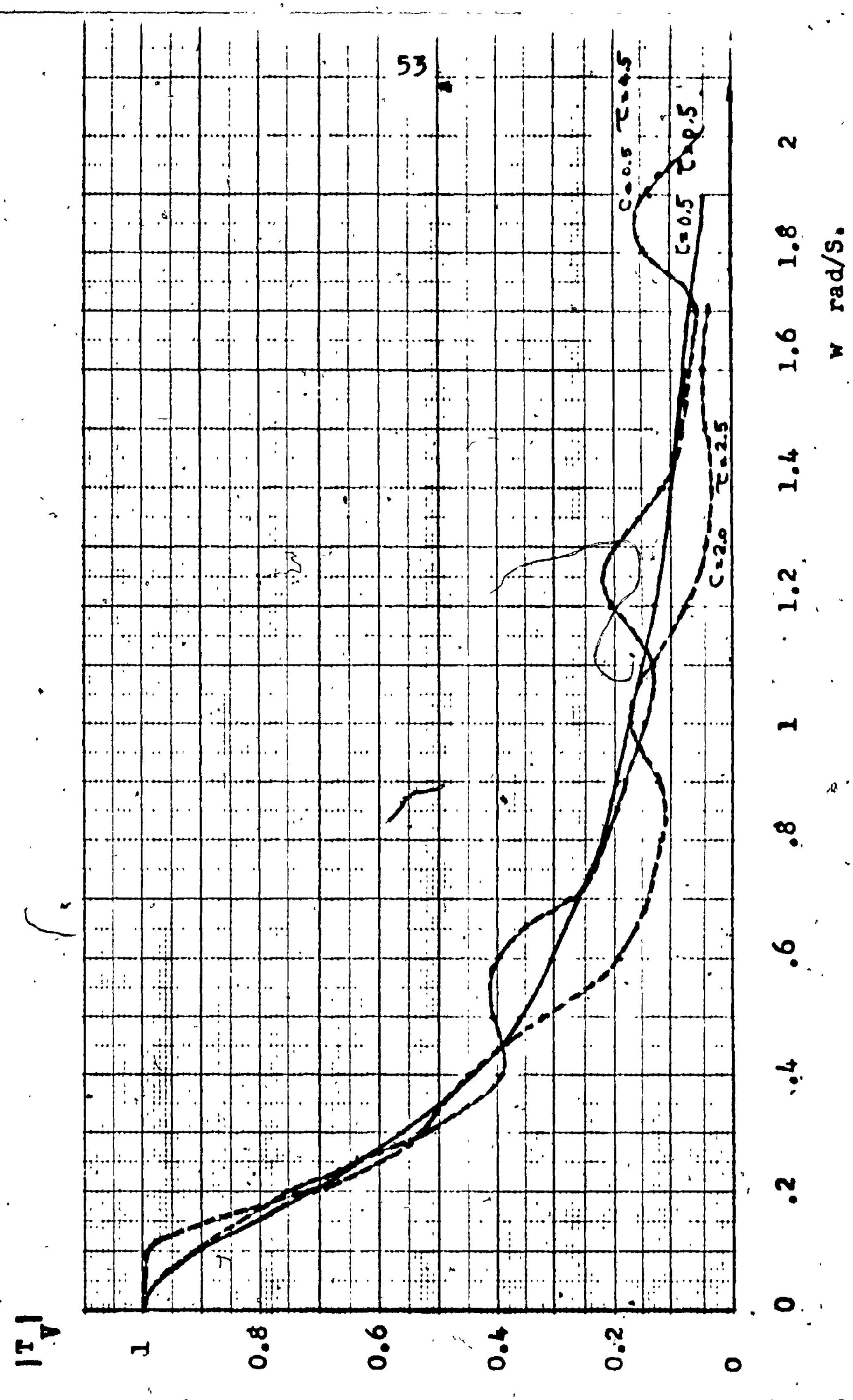

Figure 2.2.21 **Response of the low-pass filter for L_1=5, L_2=0.5 C=0.5 to 2 and τ=0.5 to 2.5 .**

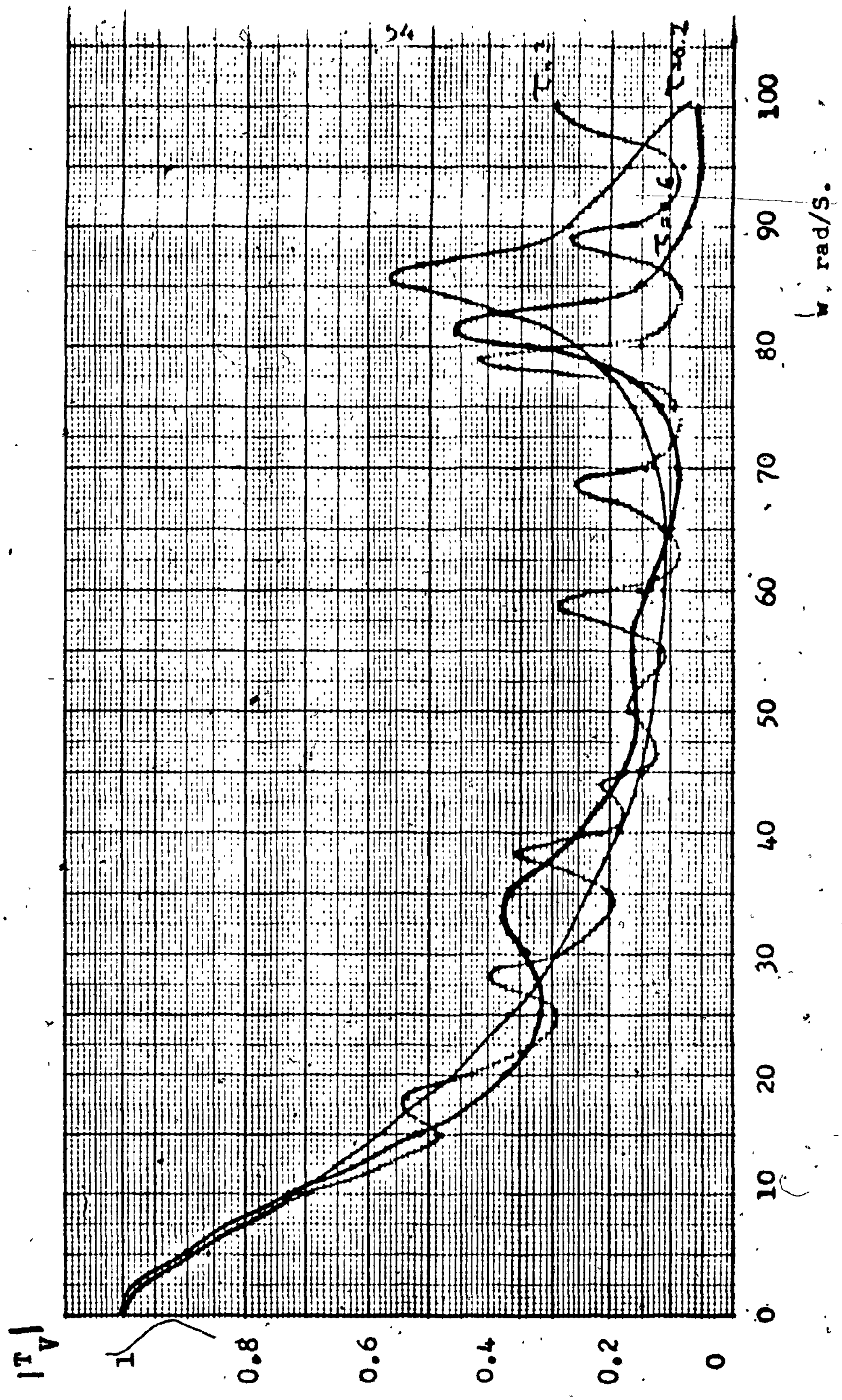

Figure 2.2.22 Response of the low-pass filter for $L_1=1, L_2=0.1$, $C=0.1$ and $\tau=0.2$ to 3 .

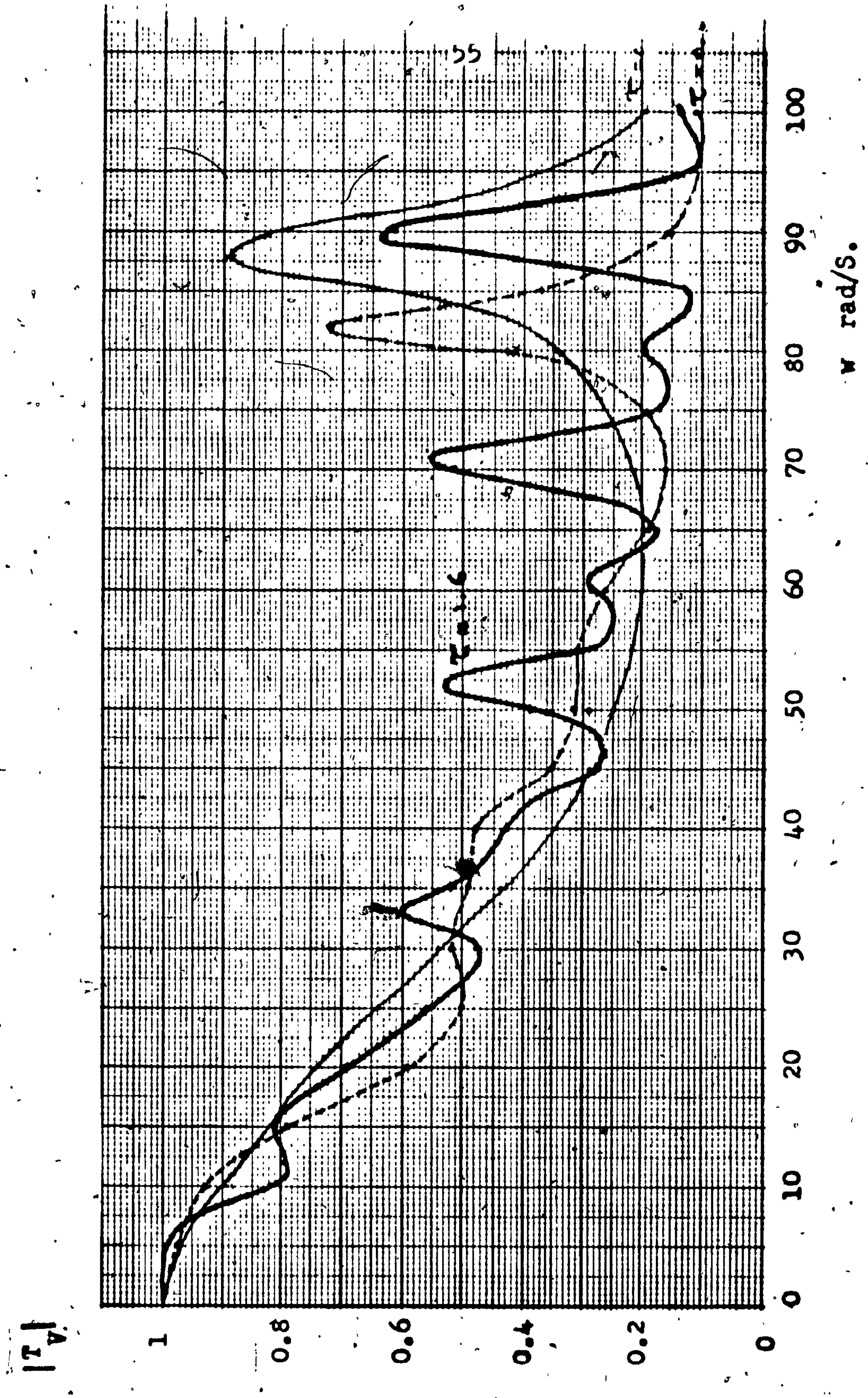

Figure 2.2.23 Response of Low-Pass Filter for L_1=0.5, L_2=0.1, C=0.1 and τ =0.2 to 1.6

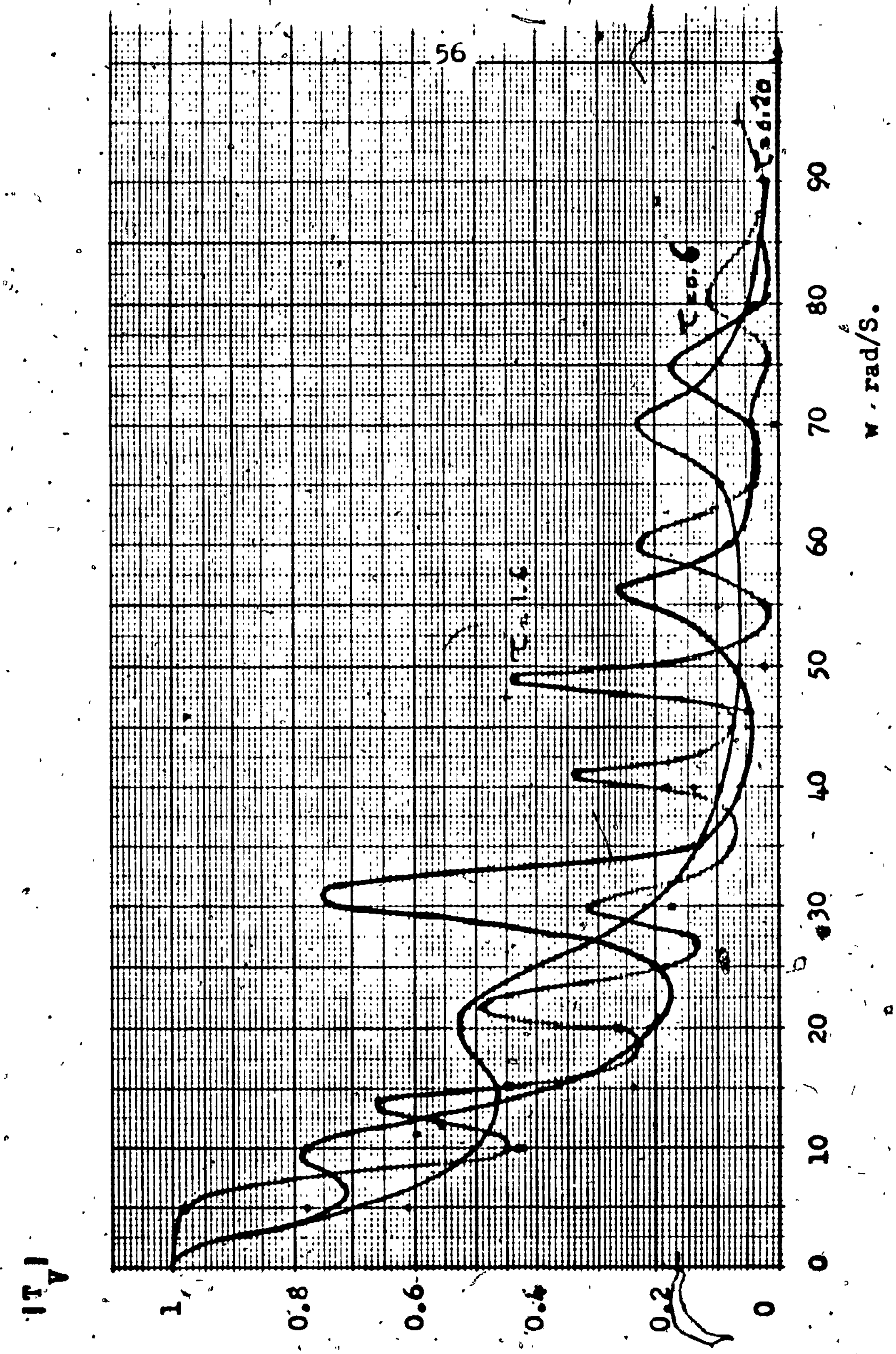

Figure 2.2.24 Response of Low-Pass Filter for L_1=0.5, L_2=1.5, C=0.1 and τ =0.2 to 1.6

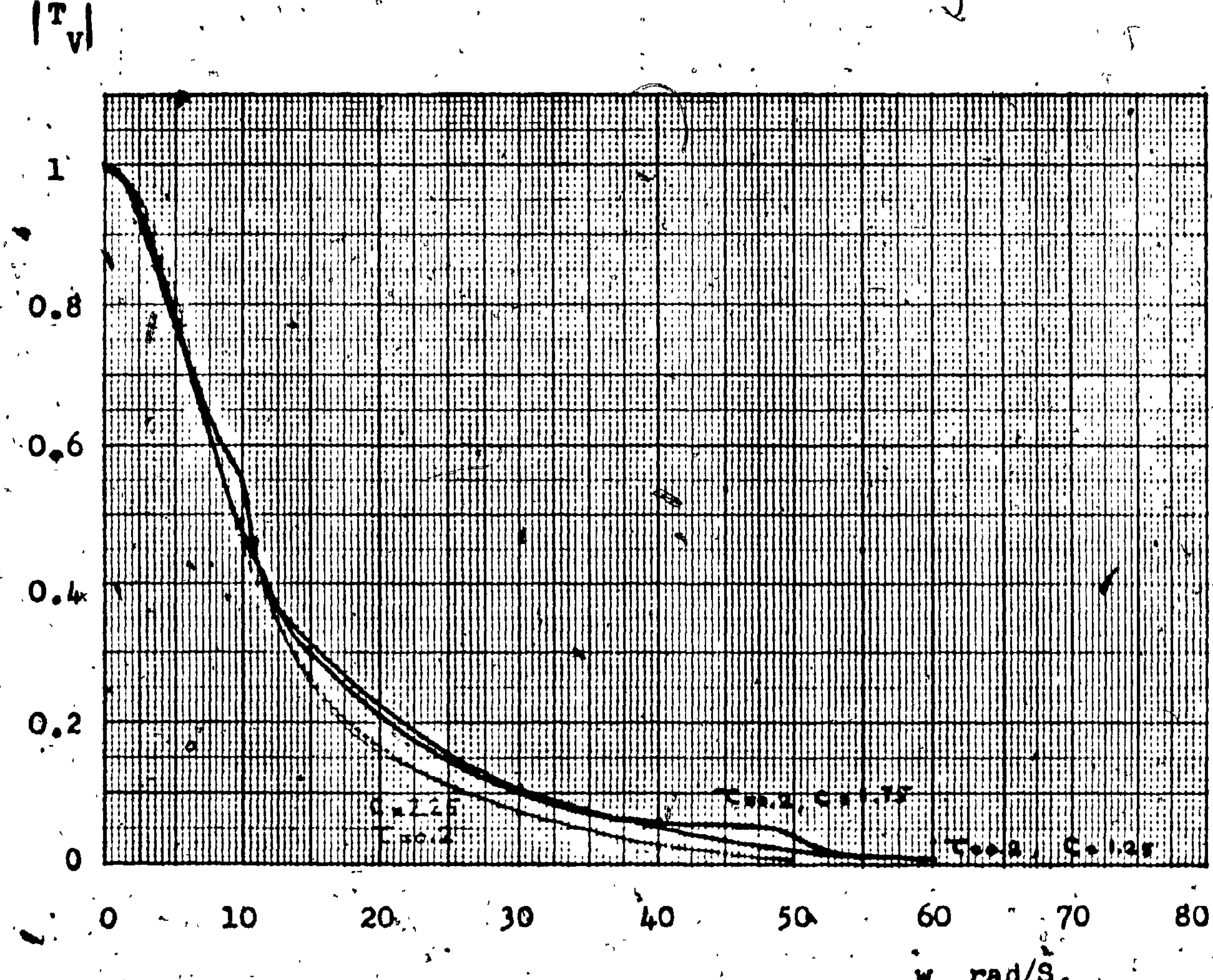

Figure 2.2.25 Response of Low-Pass Filter for L_1=0.5, L_2=1.5, C=1.5 and 2.5 and τ =0.2 to 1.6

CHAPTER III

A HIGH-PASS MIXED LUMPED-DISTRIBUTED FILTER

3.1 Introduction

In this chapter, we shall consider the design of a high-pass filter. The lumped units consist of two capacitances and an inductance. The distributed units consist of two U.E.'s. The entire network is terminated in a one ohm resistance.

3.2 Analysis of the Network

The considered network is shown in figure 3.1. The transfer function is found using the chain matrix method as follows:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{SC_1} \\ 0 & 1 \end{bmatrix} \qquad (3.1)$$

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix} = \begin{bmatrix} \cosh \gamma 1 & \sinh \gamma 1 \\ \sinh \gamma 1 & \cosh \gamma 1 \end{bmatrix} \qquad (3.2)$$

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{SL} & 1 \end{bmatrix} \qquad (3.3)$$

$$\begin{bmatrix} A_5 & B_5 \\ C_5 & D_5 \end{bmatrix} = \begin{bmatrix} (1+\frac{1}{SC_2}) & \frac{1}{SC_2} \\ 1 & 1 \end{bmatrix} \qquad (3.4)$$

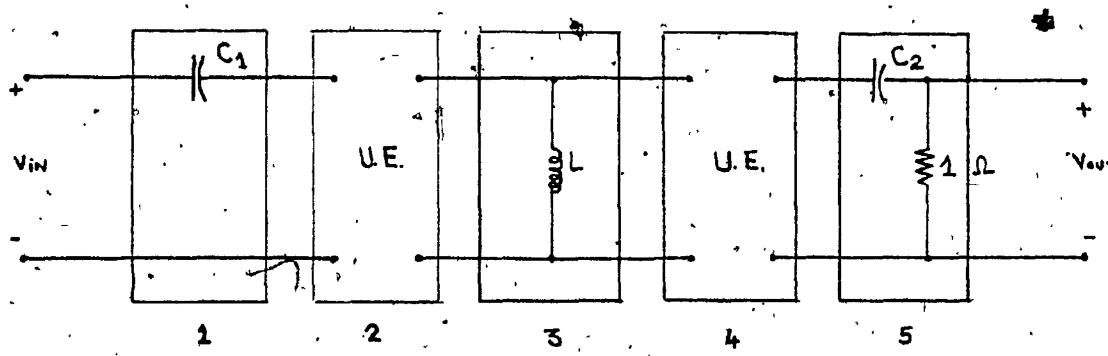

Figure 3.1. Network consisting of five sections. Two capacitive reactances, one inductive reactance and two unit elements. The network is terminated in a one ohm resistance.

Where γ = the propagation constant of the line, and

γl = the electrical length of the line, and

$\gamma l = S\tau$.

The overall chain matrix is the successive multiplication of the five matrices. The overall A parameter is the inverse of the transfer function. Substituting equations (3.1, 3.2, 3.3 and 3.4) in equation (2.5) we get:

$$A_{OVERALL} = \cosh^2(S\tau)\left(1+\frac{1}{SC_2}+\frac{1}{S^2LC_1}+\frac{1}{S^3LC_1C_2}+\frac{1}{SC_1}\right)$$
$$+\sinh^2(S\tau)\left(1+\frac{1}{SL}+\frac{1}{SC_2}+\frac{2}{SC_1}+\frac{1}{S^2\,C_1C_2}\right)$$
$$+\sinh(S\tau)\cosh(S\tau)\left(3+\frac{1}{SL}+\frac{1}{SC_2}+\frac{3}{SC_1}+\frac{1}{S^2LC_2}\right.$$
$$\left.+\frac{1}{S^2LC_1}+\frac{2}{S^2C_1C_2}\right) \qquad (3.5)$$

Substituting :

$S\tau = jw$,and

$$\cosh^2(S\tau) = \frac{1}{2}+\frac{1}{2}\cos(2w\tau), \text{ and}$$
$$\sinh^2(S\tau) = -\frac{1}{2}+\frac{1}{2}\cos(2w\tau), \text{ and}$$
$$\sinh(S\tau)\cosh(S\tau) = \frac{j}{2}\sin(2w\tau). \qquad (3.6)$$

We get:

$$A_{OVERALL} = F + jG \qquad (3.7)$$

and

$$|T_V| = \frac{1}{\sqrt{F^2+G^2}} \qquad (3.8)$$

Where:

$$F = \frac{1}{2w^2}\left(\frac{-1}{LC_1} + \frac{1}{C_1C_2}\right) + \frac{1}{2}\cos(2w\tau)\left(2 - \frac{1}{w}\left(\frac{1}{LC_1} + \frac{1}{C_1C_2}\right)\right)$$

$$+ \frac{1}{2}\sin(2w\tau)\left(\frac{1}{w}\left(\frac{1}{L} + \frac{1}{C_1} + \frac{1}{C_2}\right)\right) \qquad (3.9)$$

and

$$G = \frac{1}{2}\frac{1}{w}\left(\frac{1}{L} + \frac{1}{C_1}\right) + \frac{1}{2}\frac{1}{w^3}\frac{1}{LC_1C_2}$$

$$- \cos(2w\tau)\left(\frac{1}{2w}\left(\frac{3}{C_1} + \frac{2}{C_2} + \frac{1}{L}\right) - \frac{1}{w^3 LC_1C_2}\right)$$

$$+ \sin(2w\tau)\left(\frac{3}{2} - \frac{1}{w^2}\left(\frac{1}{LC_1} + \frac{1}{LC_2} + \frac{2}{C_1C_2}\right)\right) \qquad (3.10)$$

Therefore:

$$F^2 = \frac{1}{2} + \frac{1}{w^2}\left(\frac{1}{8L^2} + \frac{9}{8C_1^2} + \frac{1}{8C_2^2} + \frac{1}{4LC_1} + \frac{1}{4LC_2} + \frac{1}{4C_1C_2}\right)$$

$$+ \frac{1}{w^4}\left(\frac{5}{8C_1^2C_2^2} + \frac{5}{8L^2C_1^2} - \frac{1}{4LC_1^2C_2}\right)$$

$$+ \cos(2w\tau)\left(\frac{1}{w^2}\left(\frac{1}{C_1C_2} - \frac{1}{LC_1}\right) + \frac{1}{w^4}\left(\frac{1}{2L^2C_1^2} - \frac{1}{2C_1^2C_2^2}\right)\right)$$

$$+ \operatorname{Sin}(2w\tau)\left(\frac{1}{w^3}\right)\left(\frac{2}{C_1^2 C_2} - \frac{2}{LC_1^2}\right)$$

$$+\operatorname{Cos}(4w\tau)\left(\frac{1}{2} - \left(\frac{1}{w^2}\right)\left(\frac{1}{4LC_1} + \frac{1}{4C_1C_2} + \frac{1}{4LC_2} + \frac{1}{8L^2} + \frac{1}{8C_2^2} + \frac{9}{8C_1^2}\right)\right.$$

$$\left. + \frac{1}{w^4}\left(\frac{1}{8L^2C_1^2} + \frac{1}{C_1^2C_2^2} + \frac{2}{LC_1^2C_2}\right)\right)$$

$$+\operatorname{Sin}(4w\tau)\left(\frac{1}{w}\left(\frac{1}{2L} + \frac{3}{2C_1} + \frac{1}{2C_2}\right)\right.$$

$$\left. - \frac{1}{w^3}\left(\frac{1}{4L^2C_1} + \frac{3}{4LC_1^2} + \frac{1}{2LC_1C_2} + \frac{3}{4C_1^2C_2^2} + \frac{1}{4C_1C_2^2}\right)\right) \tag{3.11}$$

and

$$G^2 = \frac{1}{w^2}\left(\frac{5}{8L^2} + \frac{13}{8C_1^2} + \frac{1}{2C_2^2} + \frac{3}{2C_1C_2} + \frac{3}{4LC_1} + \frac{1}{2LC_2}\right) + \frac{9}{8}$$

$$+ \frac{1}{w^4}\left(\frac{1}{4L^2C_1C_2} - \frac{1}{4LC_1^2C_2} - \frac{1}{2LC_1C_2^2}\right) + \frac{5}{8w^6L^2C_1^2C_2^2}$$

$$+ \operatorname{Cos}(2w\tau)\left(\frac{1}{w^2}\left(\frac{4}{LC_1} + \frac{2}{LC_2} + \frac{1}{L^2} + \frac{3}{C_1^2} + \frac{2}{C_1C_2}\right)\right.$$

$$+ \frac{1}{w^4}\left(\frac{2}{LC_1^2C_2} + \frac{2}{LC_1C_2^2}\right) - \frac{1}{w^6L^2C_1^2C_2^2}\Bigg)$$

$$+ \operatorname{Sin}(2w\tau)\left(\frac{3}{2wL} + \frac{3}{2wC_1} - \frac{1}{w^3}\left(\frac{1}{2LC_1C_2} + \frac{1}{L^2C_1} + \frac{1}{2L^2C_2} + \frac{1}{C_1^2C_2}\right)\right.$$

$$\left. - \frac{1}{w^5}\left(\frac{1}{2L^2C_1^2C_2} + \frac{1}{2L^2C_1C_2^2} + \frac{1}{LC_1^2C_2^2}\right)\right)$$

$$+ \operatorname{Cos}(4w\tau)\left(\frac{-9}{8} + \frac{1}{w^2}\left(\frac{9}{8C_1^2} + \frac{1}{2C_2^2} + \frac{1}{8L^2} + \frac{7}{4C_1C_2} + \frac{7}{8LC_1}\right.\right.$$

$$\left. + \frac{5}{8LC_2}\right) - \frac{1}{w^4}\left(\frac{5}{4LC_1^2C_2} + \frac{1}{2L^2C_1C_2} + \frac{1}{8L^2C_1^2} + \frac{1}{8L^2C_2^2} + \frac{1}{8C_1^2C_2^2}\right.$$

$$\left.\left. + \frac{1}{LC_1C_2^2}\right) + \frac{1}{w^6}\ \frac{1}{8L^2C_1^2C_2^2}\right)$$

$$+ \operatorname{Sin}(4w\tau)\left(\frac{1}{w}\left(\frac{1}{4C_1} + \frac{3}{2C_2} + \frac{3}{4L}\right)\right.$$

$$- \frac{1}{w^3}\left(\frac{5}{2LC_1C_2} + \frac{1}{4L^2C_1} + \frac{1}{4L^2C_2} + \frac{1}{4LC_1^2} + \frac{1}{2LC_2^2} + \frac{1}{C_1C_2^2}\right)$$

$$\left. + \frac{1}{w^5}\left(\frac{1}{4L^2C_1^2C_2} + \frac{1}{4L^2C_1C_2^2} + \frac{1}{2LC_1^2C_2^2}\right)\right) \qquad (3.12)$$

or

$$\left| T_V \right| = 1 \,/\, \mathrm{SQRT}\left(\frac{13}{8} - \frac{5}{8} \mathrm{Cos}\,(4w\tau) \right.$$

$$+ \frac{1}{w}\left(\left(\frac{3}{2L} + \frac{3}{2C_1} \right) \mathrm{Sin}\,(2w\tau) \right.$$

$$\left. + \left(\frac{5}{4\,L} \quad \frac{7}{4\,C_1} \quad \frac{2}{C_2} \right) \mathrm{Sin}\,(4w\tau) \right)$$

$$+ \frac{1}{w^2}\left(\left(\frac{3}{4\,L^2} + \frac{21}{8\,C_1^2} + \frac{5}{8\,C_2^2} + \frac{7}{4C_1C_2} + \frac{1}{LC_1} + \frac{3}{4LC_2} \right) \right.$$

$$+ \left(\frac{3}{LC_1} + \frac{2}{LC_2} + \frac{1}{L^2} + \frac{3}{C_1^2} + \frac{3}{C_1C_2} \right) \mathrm{Cos}\,(2w\tau)$$

$$\left. + \left(\frac{3}{8\,C_2^2} + \frac{3}{2C_1C_2} + \frac{3}{2LC_1} + \frac{1}{2LC_2} \right) \mathrm{Cos}\,(4w\tau) \right)$$

$$+ \frac{1}{w^3}\left(\left(\frac{2}{C_1C_2^2} - \frac{1}{C_1^2C_2} - \frac{2}{LC_1^2} - \frac{2}{LC_1C_2} - \frac{1}{L^2C_1} - \frac{1}{2L^2C_2} \right) \right.$$

$$\mathrm{Sin}\,(2w\tau)$$

$$- \left(\frac{1}{2L^2C_1} + \frac{3}{2LC_1^2} + \frac{3}{LC_1C_2} + \frac{3}{4C_1^2C_2} + \frac{1}{4C_1C_2^2} + \frac{1}{2LC_2^2} \right.$$

$$+ \frac{1}{C_1 C_2^2}) \text{ Sin } (4w\tau))$$

$$+ \frac{1}{w^4} ((\frac{5}{8C_1^2 C_2^2} + \frac{5}{8L^2 C_1^2} - \frac{1}{2LC_1^2 C_2} + \frac{1}{4L^2 C_1 C_2} - \frac{1}{2LC_1 C_2^2})$$

$$+ (\frac{2}{LC_1^2 C_2} + \frac{2}{LC_1 C_2^2} + \frac{1}{2L^2 C_1^2} - \frac{1}{2C_1^2 C_2^2}) \text{ Cos } (2w\tau)$$

$$+ (\frac{7}{8C_1^2 C_2^2} + \frac{3}{4LC_1^2 C_2} - \frac{1}{LC_1 C_2^2} - \frac{1}{2L^2 C_1 C_2} - \frac{1}{8L^2 C_2^2})$$

$$\text{Cos } (4w\tau)$$

$$+ \frac{1}{w^5} (- (\frac{1}{2L^2 C_1^2 C_2} + \frac{1}{2L^2 C_1 C_2^2} + \frac{1}{LC_1^2 C_2^2}) \text{ Sin } (2w\tau)$$

$$+ (\frac{1}{4L^2 C_1^2 C_2} \quad \frac{1}{4L^2 C_1 C_2^2} \quad \frac{1}{2LC_1^2 C_2^2}) \text{ Sin } (4w\tau))$$

$$+ \frac{1}{w^6} (\frac{5}{8L^2 C_1^2 C_2^2} - (\frac{1}{L^2 C_1^2 C_2^2} \text{ Cos } (2w\tau))$$

$$+ \frac{1}{8L^2 C^2 C^2} \text{ Cos } (4w\tau))' \qquad (3.13)$$

In order to have a high-pass filter the following condition must be fulfilled:

$$\left| T_V \right| \leqslant 1 \qquad (3.14)$$

Analytical solution of equation (3.14) is difficult because it is a sixth order equation with four variables. A simplification can be considered by letting $C_1 = C_2 = L/K$. Therefore equation (3.14) becomes:

$$\frac{5}{8} - \frac{5}{8}\,\mathrm{Cos}\,(4w\tau) + \frac{1}{w}\left(\left(\frac{3}{2C}\left(1+\frac{1}{K}\right) \mathrm{Sin}\,(2w\tau)\right) + \frac{5}{4C}\left(3+\frac{1}{K}\right)\mathrm{Sin}\,(4w\tau) \right)$$

$$+\frac{1}{w^2C^2}\left(\left(5+\frac{7}{4K}+\frac{3}{4K^2}\right) + \left(6+\frac{5}{K}+\frac{1}{K^2}\right)\mathrm{Cos}\,(2w\tau) + \left(\frac{15}{8}+\frac{2}{K}\right)\mathrm{Cos}\,(4w\tau)\right)$$

$$+\frac{1}{w^3C^3}\left(\left(1-\frac{4}{K}-\frac{3}{2K^2}\right)\mathrm{Sin}\,(2w\tau) - \left(2+\frac{5}{K}+\frac{1}{2K^2}\right)\mathrm{Sin}\,(4w\tau)\right)$$

$$+\frac{1}{w^4C^4}\left(\left(\frac{5}{8}-\frac{1}{K}+\frac{7}{8K^2}\right) + \left(-\frac{1}{2}+\frac{4}{K}+\frac{1}{2K^2}\right)\mathrm{Cos}\,(2w\tau)\right.$$

$$\left. + \left(\frac{7}{8}-\frac{1}{4K}+\frac{5}{8K^2}\right)\mathrm{Cos}\,(4w\tau)\right)$$

$$+\frac{1}{w^5C^5}\left(-\frac{1}{K}\,\mathrm{Sin}\,(2w\tau) + \left(\frac{1}{2K^2}+\frac{1}{2K}\right)\mathrm{Sin}\,(4w\tau)\right)$$

$$+\frac{1}{w^6 C^6}\left(\frac{5}{8K^2}-\frac{1}{K^2}\cos(2w\tau)+\frac{1}{8K^2}\cos(4w\tau)\right)\leqslant 0 \qquad (3.15)$$

Equation (3.15) is complicated and analytical solution is difficult. The numerical solution is considered and a computer program is written to solve it.

Now we can consider some special cases as well as the general case.

3.2.1 The Lumped Section form a Constant K Filter

For this case, we have the relationship $C_1=C_2=C$, and $L=KC$. This case has been considered in equation (3.15). The computer program is given in Table 3.1 . There are three variables to adjust, namely K, C and τ . This yields families of curves which are shown in figure 3.3.1 to 3.3.5

Figure 3.2.1 shows the response for small K. We notice that the rise in the response is very slow and a far cut-off appears.

Figure 3.2.2 gives the response for medium K (K=1) and different values of capacitance. At C=0.1 , we notice that rise in response is very slow even for different values of τ . In creasing the value of C makes the rise faster and oscillation appear depending on the value of τ . At τ=1.1 oscillations happen more frequently. This is explained by the reflections in the transmission line.

Figure 3.2.3 gives the response for K=1, C=0.5 and τ=1.

We notice that oscillations take place with large difference between maxima and minima.

Figure 3.2.4 gives a family of curves for K=1 and different values of C and τ . We coclude that the increase in the value of the capacitance leads to faster rise in the response.

```
      PROGRAM FAROUK ( INPUT,OUTPUT)
C     STUDY OF A H.P.F. USING MIXED LUMPED-DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=1.
      C=0.1
33    X(1)=0.1
      DO 100 I=1,100
      H=1./A
      R=2.*X(I)*Y
      B=COS(R)
      D=SIN(R)
      F=1.0/(X(I)*C)
      G=F**2
      P=F**3
      REAL=0.5*G*(1.-H)
     1 +0.5*B*(2.-G*(1.+H))
     1 +0.5*D*(F*(4.+H))
      RIMA=0.5*F*(1.+H)  +0.5*P*H
     1 -0.5*B*(F*(5.+H)-P*H)
     1 +0.5*D*(3.-0.5*G*(1.+H))
      S=REAL**2+RIMA**2
      T(I)=1./SQRT(S)
      IF(T(I).GT.1.) GO TO 70
      IF ( X(I).GT.10.) GO TO 70
      X(I+1) = X(I) + 0.1
100   CONTINUE
      PRINT 90,A,C,Y
90    FORMAT (/20X,"A=",F4.2,20X,"C=",F12.4,20X,"Y=",F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.1
      IF (Y.LT.5.) GO TO 33
      Y=0.0
      C=C+0.1
      IF(C.LT.3.) GO TO 33
      C=0.0
      A=A+0.1
      IF ( A. LT. 5.) GO TO 33
      STOP
      END
```

Table 3.1 Computer Program of a Mixed Lumped-Distributed Constant K High-Pass Filter

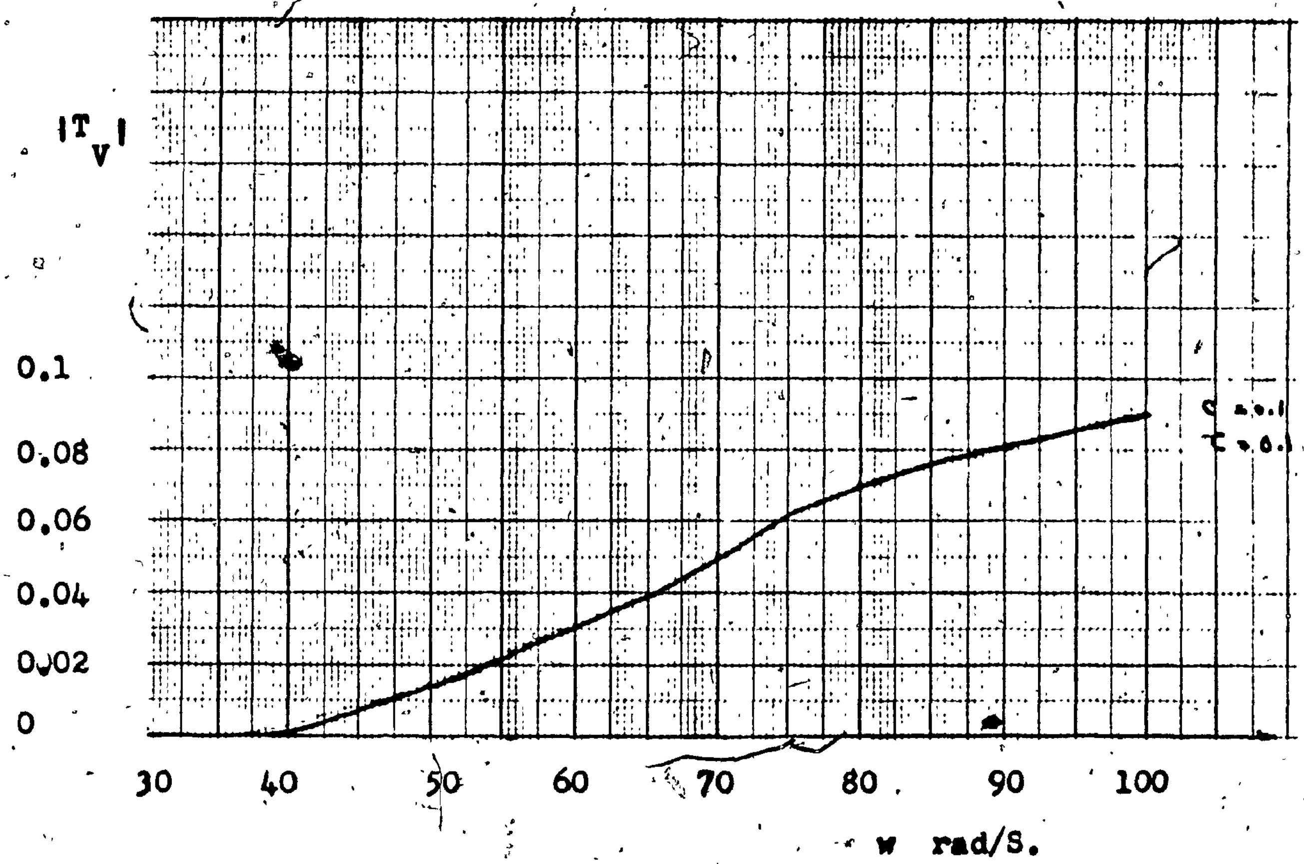

Figure 3.2.1 Response of Constant K High-Pass Filter.

K=0.1, C=0.1 and τ =0.1

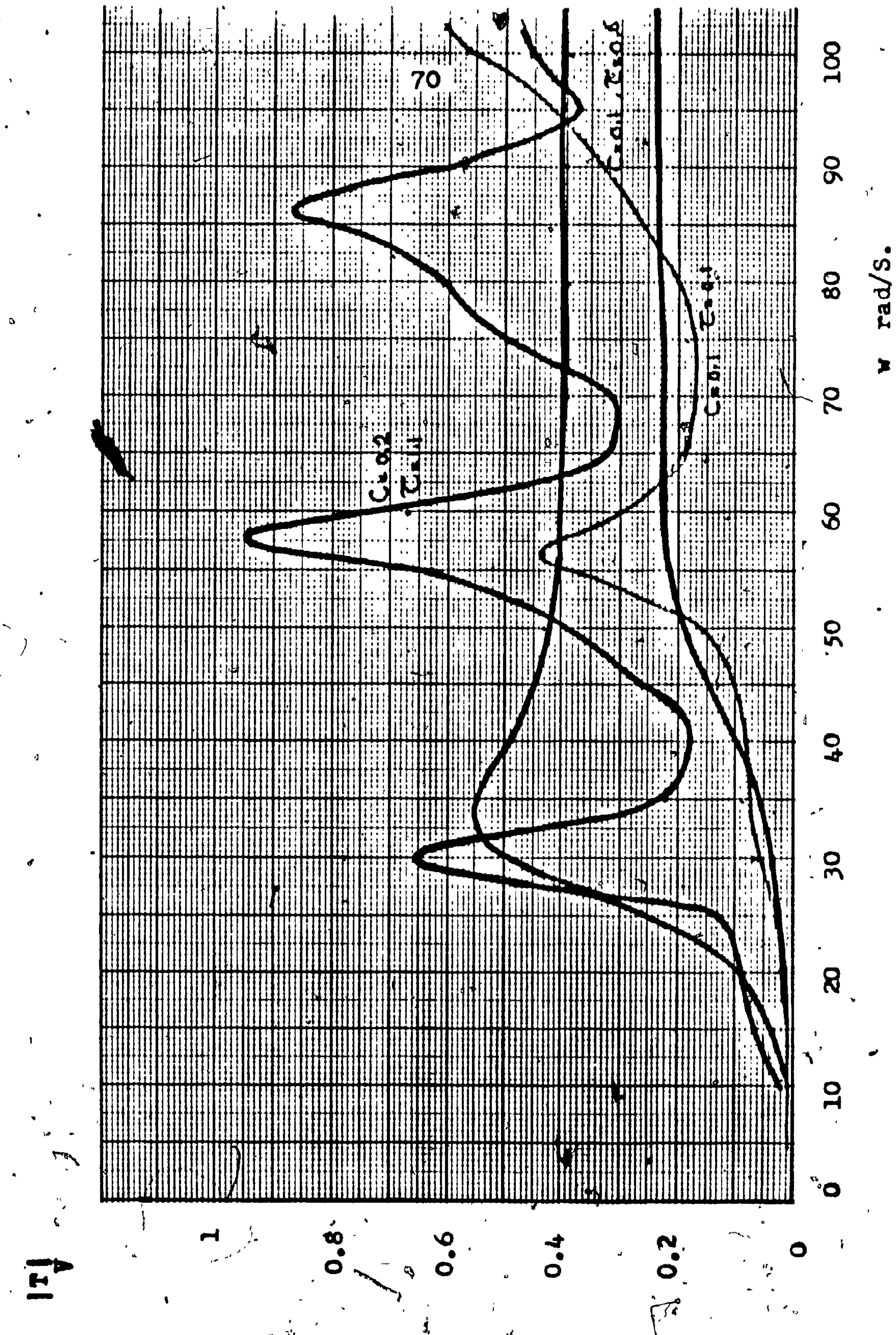

Figure 3.2.2 Response of Constant K High-Pass Filter.

K=1, C=0.5 to 0.2 and τ =0.1 to 0.6

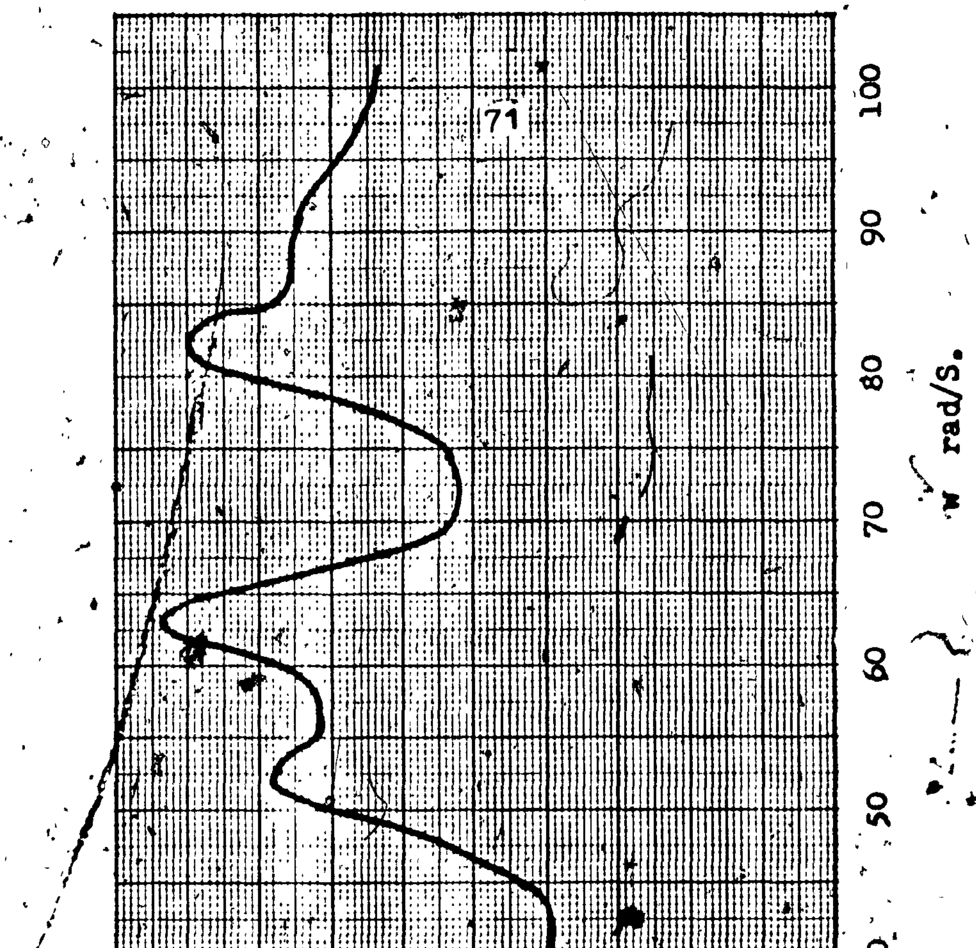

Figure 3.2.3 Response of Constant K High-Pass Filter.

K=1, ε=0.5 and τ=1

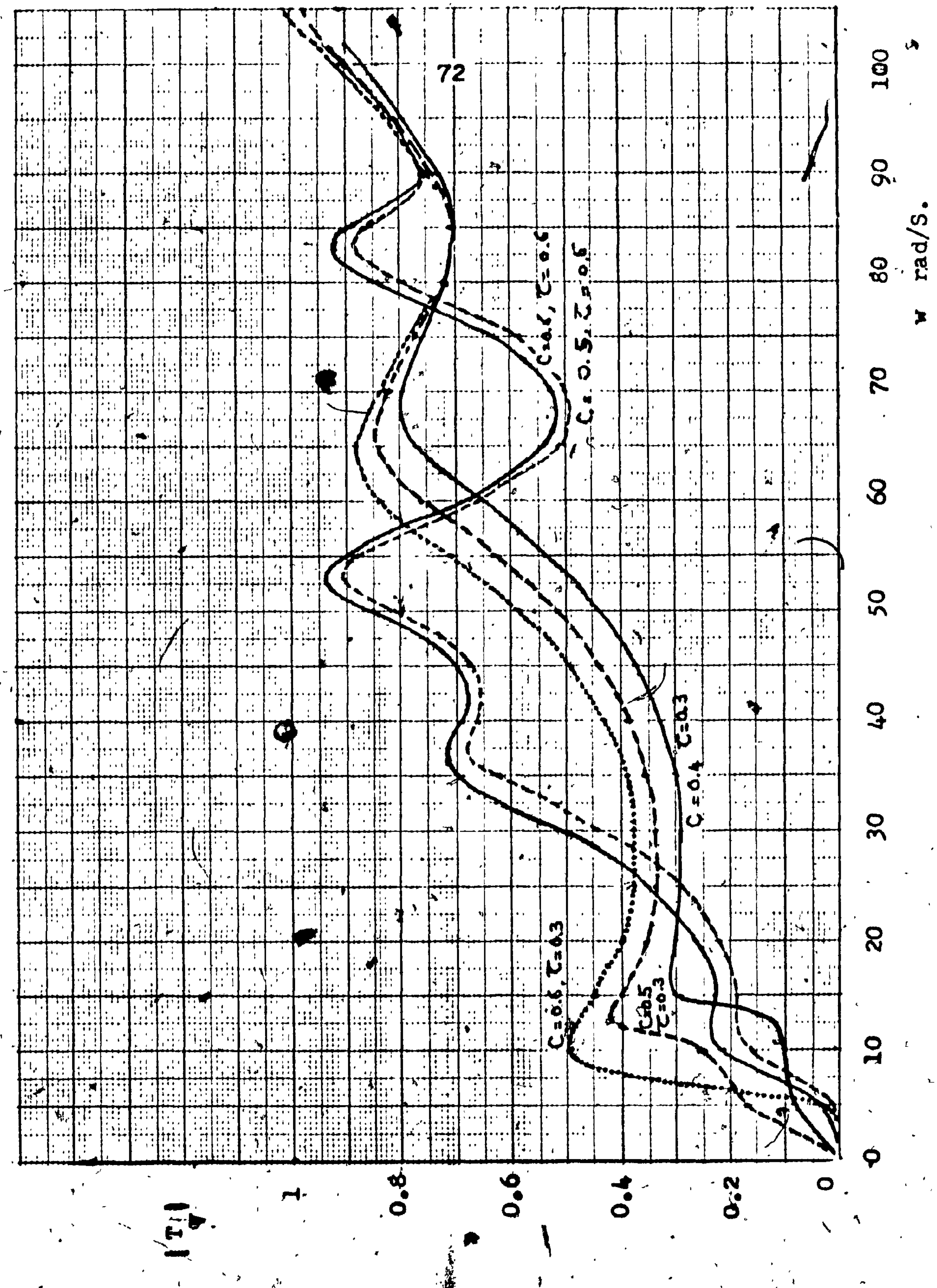

Figure 3.2.4 Response of Constant K High-Pass Filter.

K=1, C=0.2 to 0.4 and τ=0[illegible] to 3.9

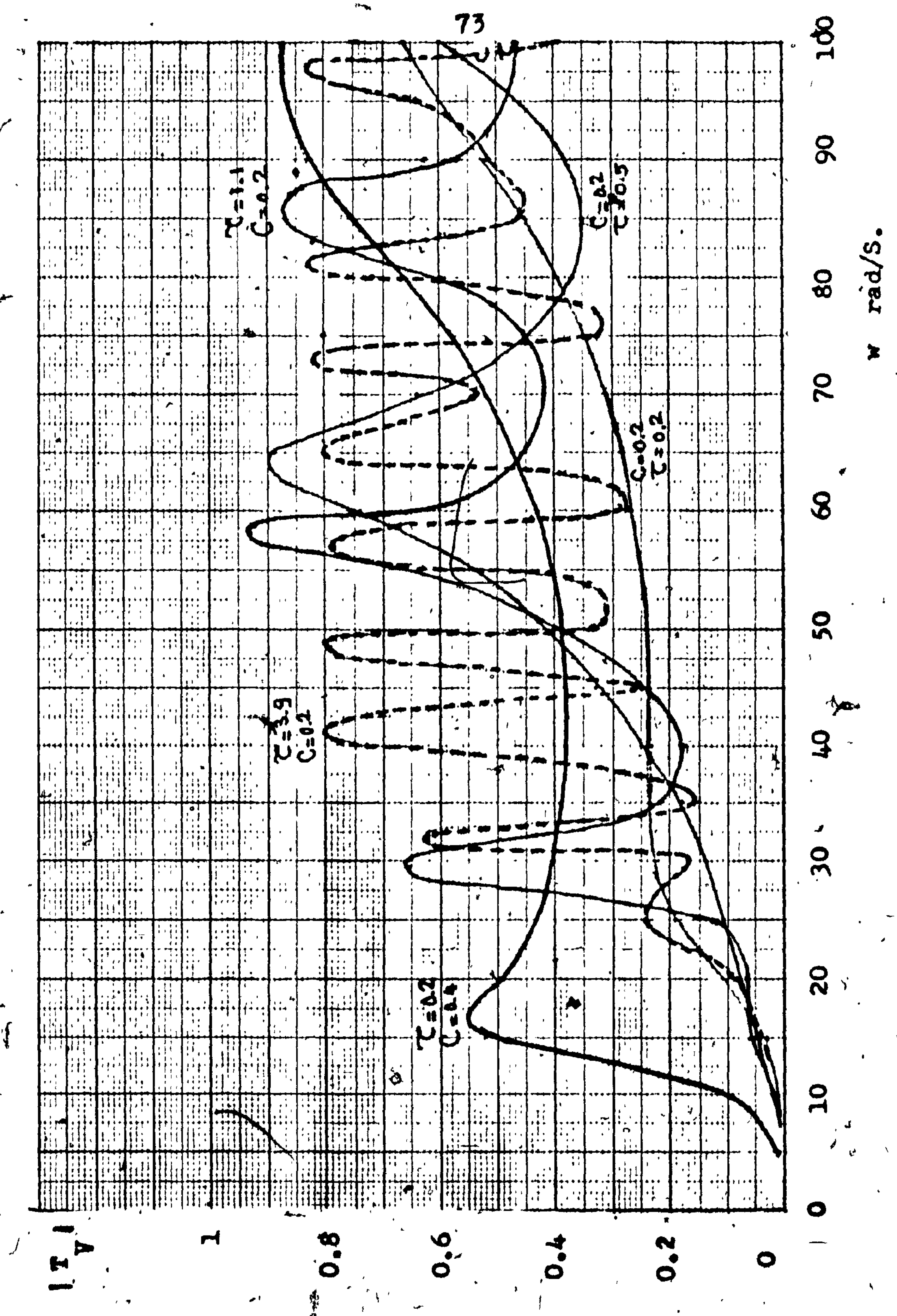

Figure 3.2.5 Response of Constant K High-Pass Filter.

K=1, C=0.2 to 0.4 and τ =0.5 to 3.9.

Figure 3.2.5 gives different responses with varying τ's. For K=1, C=0.2 and τ =0.2 , the response rises slowly. Increasing to 0.5 gives oscillations in the response. A further increase in the value of τ leads to more oscillations .

3.2.2 The Butterworth Case

The lumped sections form a Butterworth high-pass filter for which $C_1=2$, $C_2=1.33$ and L=0.66 . Hence the transfer function becomes:

$$
\begin{aligned}
T_V = 1 \,/\, \mathrm{SQRT}\Big(& \frac{13}{8} - \frac{5}{8}\,\mathrm{Cos}\,(4w\tau) \\
& + \frac{1}{w}\Big(3\,\mathrm{Sin}\,(2w\tau) + \frac{17}{4}\,\mathrm{Sin}\,(4w\tau)\Big) \\
& + \frac{1}{w^2}\Big(\frac{137}{32} + 9\,\mathrm{Cos}\,(2w\tau) + \frac{315}{128}\,\mathrm{Cos}\,(4w\tau)\Big) \\
& + \frac{1}{w^3}\Big(-\frac{67}{32}\,\mathrm{Sin}(2w\tau) - \frac{423}{128}\,\mathrm{Sin}\,(4w\tau)\Big) \\
& + \frac{1}{w^4}\Big(\frac{39}{512} + \frac{207}{128}\,\mathrm{Cos}\,(2w\tau) - \frac{171}{256}\,\mathrm{Cos}\,(4w\tau)\Big) \\
& + \frac{1}{w^5}\Big(-\frac{189}{256}\,\mathrm{Sin}\,(2w\tau) + \frac{189}{256}\,\mathrm{Sin}\,(4w\tau)\Big)
\end{aligned}
$$

$$+ \frac{1}{w^6} \left(\frac{405}{2048} - \frac{81}{128} \cos (2w\tau) + \frac{1}{1024} \cos (4w\tau) \right) \tag{3.17}$$

The computer program is given in Table 3.2 . Figure 3.2.6 gives the response for different τ's. At $\tau=0$, or no time delay, the response is a butterworth high-pass filter, at $\tau=0.1$ the effect of reflections in the U.E.'s gives oscillations. At $\tau=10$ the frequency of oscillations is much greater than for lower τ.

We can conclude that by increasing τ, the cut-off frequency moves to the right which means that it occurs at higher frequencies.

3.2.3 The Lumped Elements are Independent

The lumped sections are formed of independent elements. The transfer function is given in equation (3.12) and the computer program for this case is given in Table 3.3

Figure 3.2.7 gives the response for very low values of C_1, C_2 and L ($C_1=0.1$, $C_2=0.1$ and L=0.1). We notice that the response is rising slowly which means a far cut-off. Increase of does not introduce oscillations in the range of frequency under study (w=0 to 10).

Figure 3.2.8 shows the response for $C_1=0.1$, $C_2=0.1$ and L=2 . The response rises very slowly which means that the inductance value has no great effect on the characteristics.

Figure 3.2.9 gives the response for $C_1=0.5$, $C_2=0.5$ and L=0.5 . Small delays give far cut-off, but at $\tau=0.2$ cut-off moves right ($w_c=77$) but oscillations are important. At $\tau=0.3$ cut-off moves more to the right with incresed oscillations.

Figure 3.2.10 shows the response for $C_1=1$, $C_2=1$ and L=0.1 to 0.2 . We notice that by increasing L the rise in response is faster for a certain τ . An increase in τ produces more oscillations therefore moving the cut-off to the left.

Figure 3.2.12 shows the reponse for C_1=1, C_2=2.5 and L=0.1 . τ is choosen 0.2 . This gives an oscillation free response but cut-off is far.

Figure 3.2.13 shows the effect of increasing L to 0.5 . The rise in response is faster. An increase in the delay moves the cut-off to the left.

Figure 3.2.14 gives the response fo C_1=0.5, C_2=0.75 and L=1.5 . We notice that at τ=0.2 the response is smoother than at τ=0.5 .

3.3 Discussions

In this chapter, we have considered a particular type of mixed lumped-distributed high-pass filter. It is found that only certain values of elemental values and time delay give rise to high-pass filters.

```
      PROGRAM FAROUK ( INPUT,OUTPUT)
C     STUDY OF A H.P.F. USING MIXED LUMPED-DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=2.
      B=1.33
      C=0.66
      Y=0.0
33    X(1)=0.1
      DO 100 I=1,100
      D=1./X(I)
      E=D*D
      F=D*D*D
      G=COS(2.*X(I)*Y)
      H=SIN(2.*X(I)*Y)
      REA=0.5*E*(1./A*B-1./A*C)
     1 +0.5*G*(2.-E*(1./A*B+1./A*C))
     1 +0.5*H*D*(3./A +1./B+1./C)
      RIMA=0.5*D*(1./A+1./C)
     1 +0.5*F*1./A*B*C
     1 -0.5*G*D*(3./A+2./B+1./C)
     1 -0.5*G*F*1./A*B*C
     1 +0.5*H*(3.-E*(1./A*C+1./B*C+2./A*B))
      S=REA**2+RIMA**2
      T(I)=1./SQRT(S)
      IF(T(I).GT.1.) GO TO 70
      IF ( X(I).GT.10.) GO TO 70
      X(I+1) = X(I) + 0.1
100   CONTINUE
        PRINT 90,A,B,C,Y
90     FORMAT(/,10X,F4.2,10X,F4.2,10X,F4.2,10X,F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.1
      IF (Y.LT.5.) GO TO 33
      Y=0.0
      C=C+0.1
      IF(C.LT.3.) GO TO 33
      C=0.0
      STOP
      END
```

Table 3.2 Computer grogram of a Mixed Lumped-Distributed Butterworth High-Pass Filter

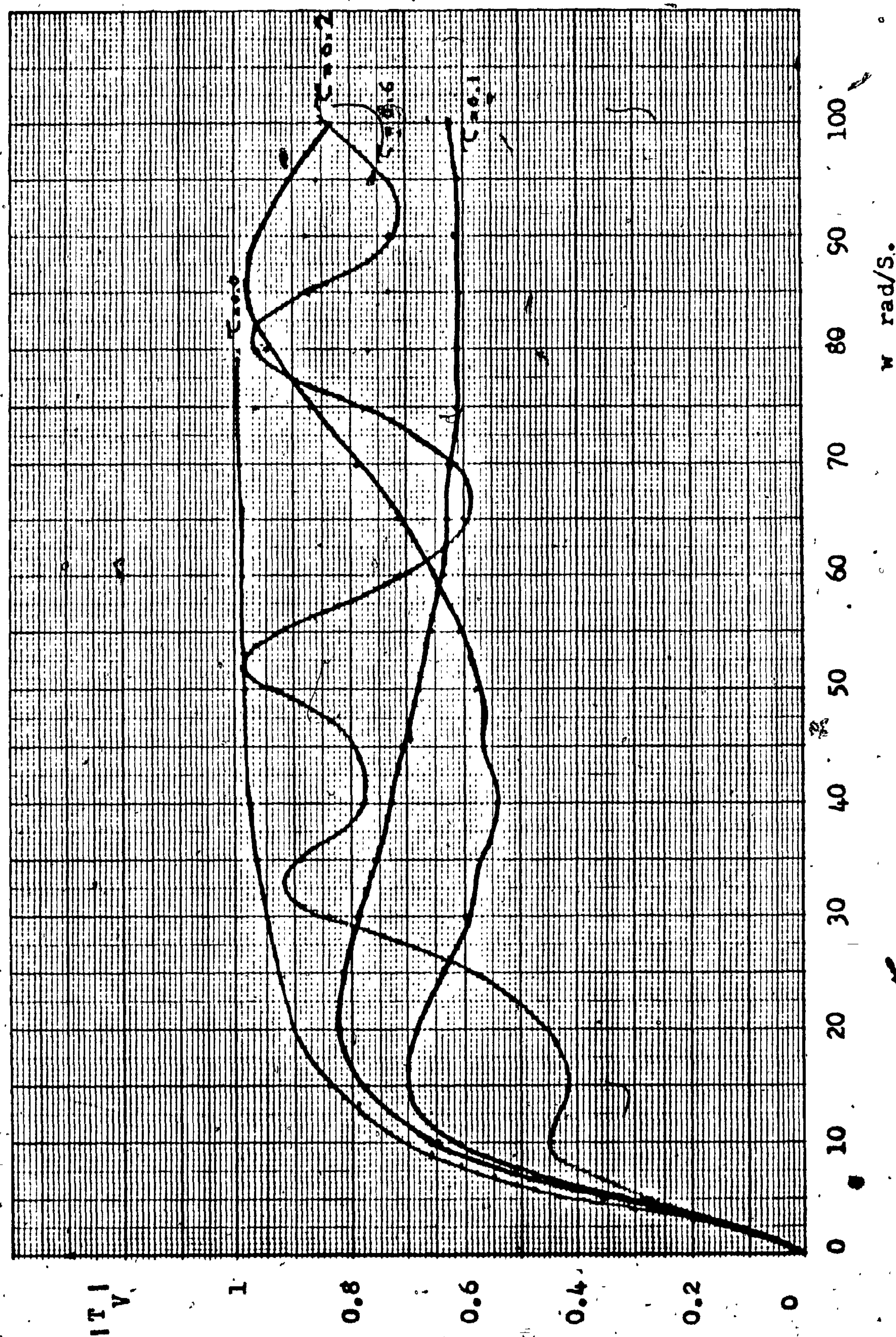

Figure 3.2.6 Response of Butterworth High-Pass Filter.

$C_1=2$, $C_2=1.33$, L=0.66 and $\tau=0.1$ to 0.6

```
      PROGRAM FAROUK ( INPUT,OUTPUT)
C     STUDY OF A H.P.F. USING MIXED LUMPED-DISTRIBUTED STRUCTURES
      DIMENSION X(101),T(101)
      A=0.5
      B=0.5
      C=0.5
      Y=0.0
33    X(1)=0.1
      DO 100 I=1,100
      D=1./X(I)
      E=D*D
      F=D*D*D
      G=COS(2.*X(I)*Y)
      H=SIN(2.*X(I)*Y)
      REA=0.5*F*(1./A*B-1./A*C)
     1 +0.5*G*(2.-E*(1./A*B+1./A*C))
     1 +0.5*H*D*(3./A +1./B+1./C)
      RIMA=0.5*D*(1./A+1./C)
     1 +0.5*F*1./A*B*C
     1 -0.5*G*D*(3./A+2./B+1./C)
     1 -0.5*G*F*1./A*B*C
     1 +0.5*H*(3.-E*(1./A*C+1./B*C+2./A*B))
      S=REA**2+RIMA**2
      T(I)=1./SQRT(S)
      IF(T(I).GT.1.) GO TO 70
      IF ( X(I).GT.10.) GO TO 70
      X(I+1) = X(I) + 0.1
100   CONTINUE
      PRINT 90,A,B,C,Y
90    FORMAT(/,10X,F4.2,10X,F4.2,10X,F4.2,10X,F4.2/)
      DO 20 I=1,10
      PRINT 91,T(I),T(I+10),T(I+20),T(I+30),T(I+40),T(I+50),
     1 T(I+60),T(I+70),T(I+80),T(I+90)
91    FORMAT (10X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4,
     1 5X,F6.4,5X,F6.4,5X,F6.4,5X,F6.4)
20    CONTINUE
70    Y=Y+0.1
      IF (Y.LT.5.) GO TO 33
      Y=0.0
      C=C+0.1
      IF(C.LT.3.) GO TO 33
      C=0.0
      STOP
      END
```

Table 3.3 Computer Program of a Mixed Lumped-Distributed Independent Elements High-Pass Filter

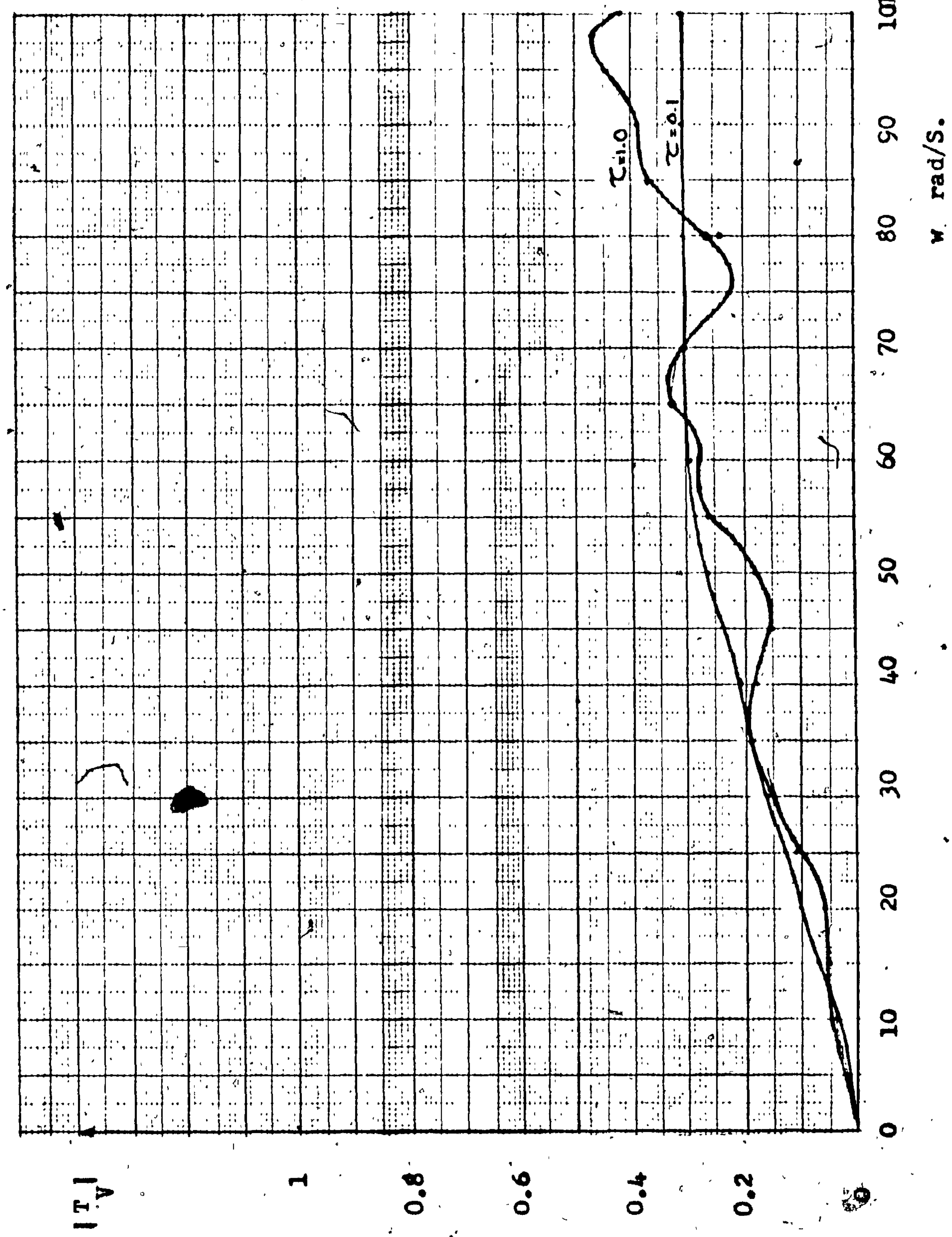

Figure 3.2.7 Response of Independent Elements Type High-Pass Filter.

C_1=0.1, C_2=0.1, L=1 and τ =0.1 to 1

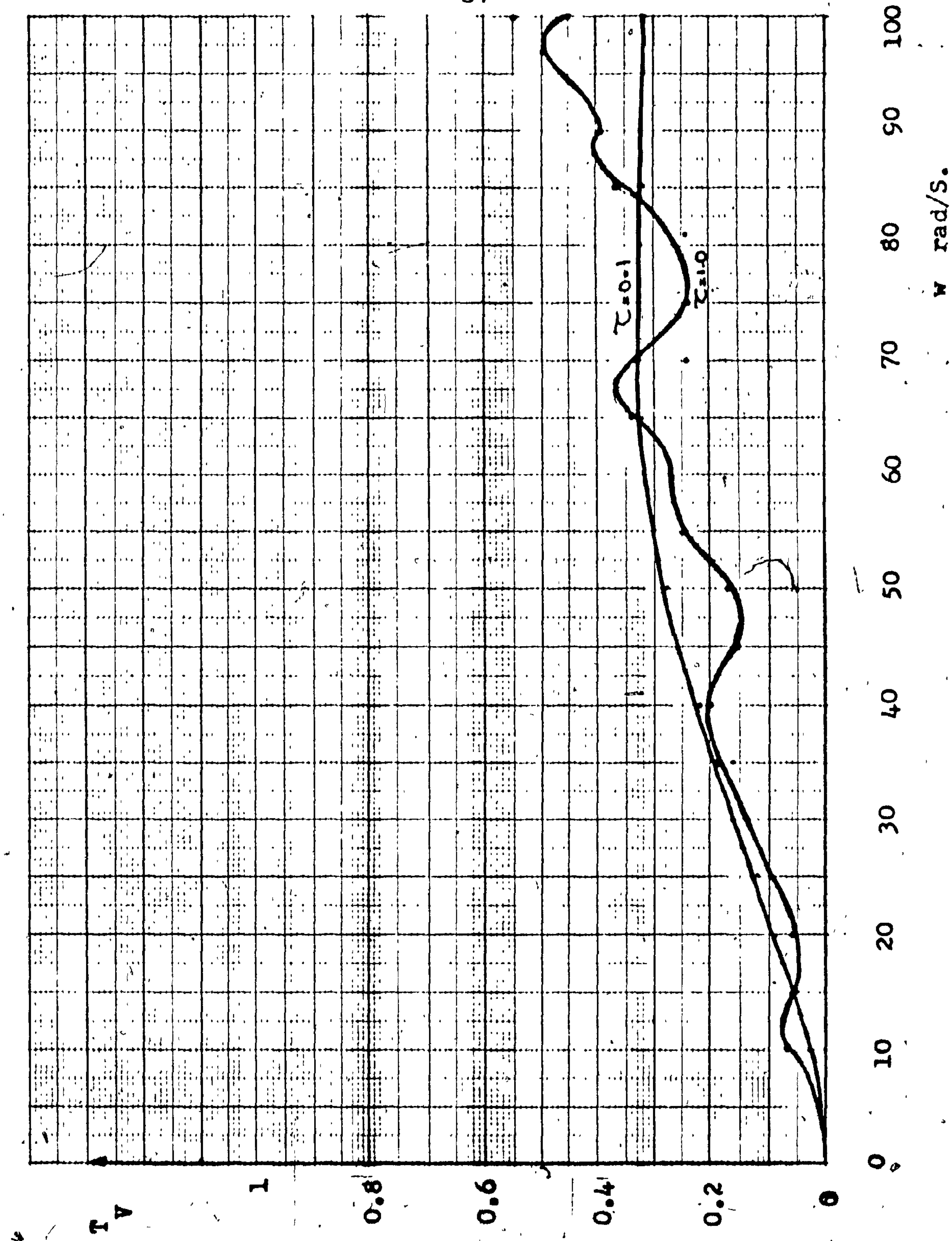

Figure 3.2.8 Response of Independent Elements Type High-Pass Filter.

C_1=0.1, C_2=0.1, L=2 and τ=0.1 to 1

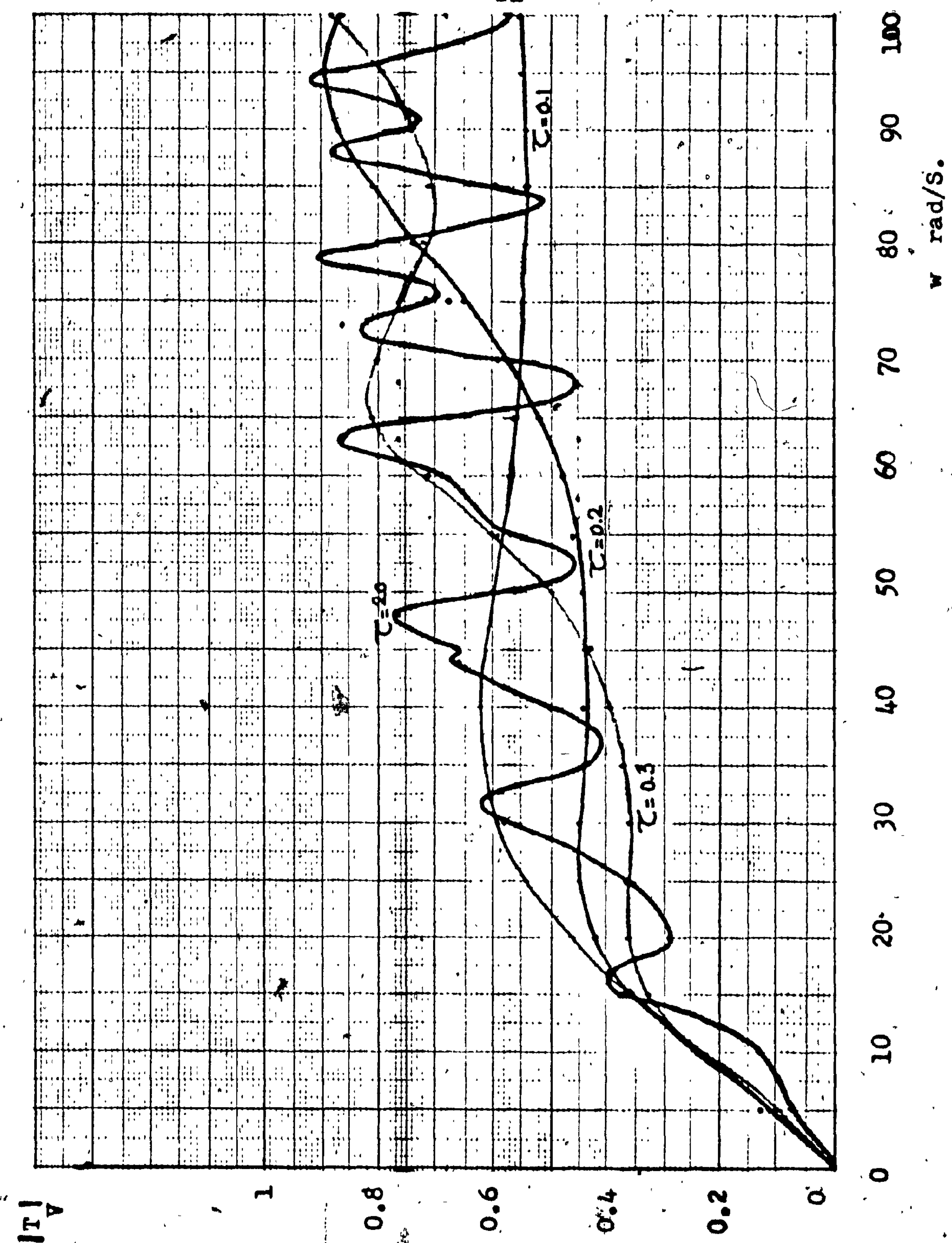

Figure 3.2.9 Response of Independent Elements Type High-Pass Filter.

C_1=0.5, C_2=0.5, L=0.5 and τ =0.1 to 2

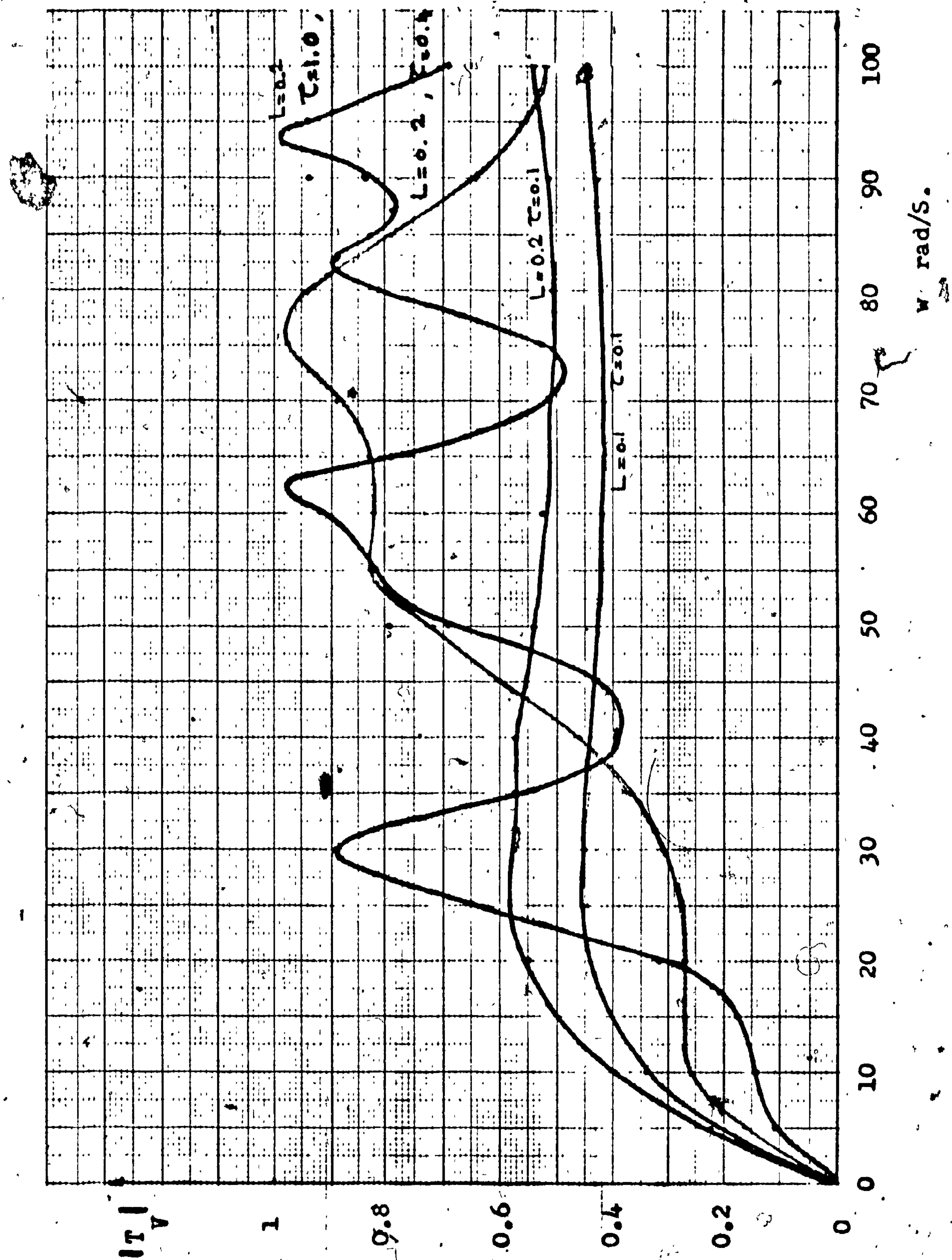

Figure 3.2.10 Response of Independent Elements Type High-Pass Filter.

$C_1=1$, $C_2=1$, $L=0.1$ to 0.2 and $\tau=0.1$ to 1

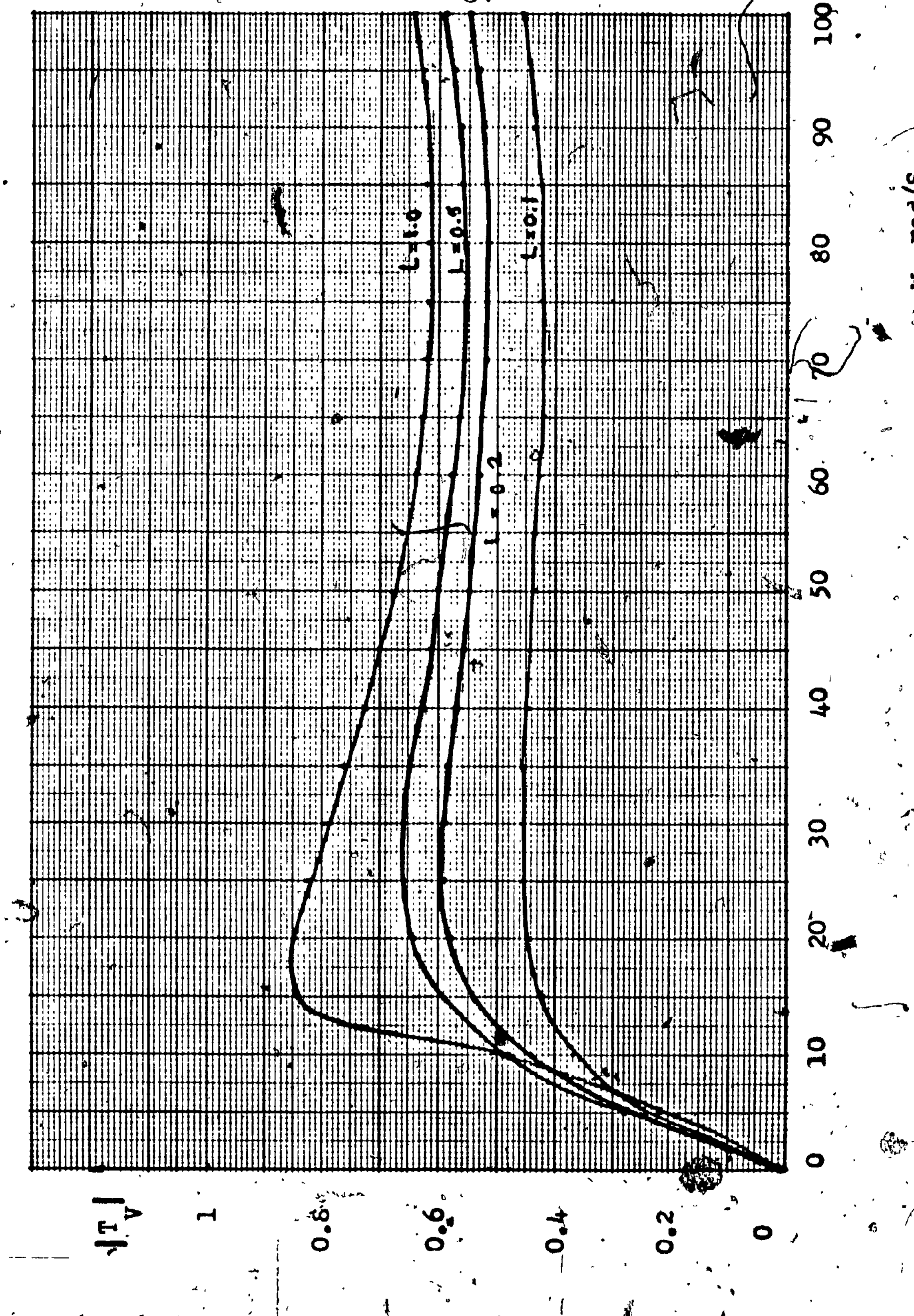

Figure 3.2.11 Response of Independent Elements Type High-Pass Filter.

$C_1=1$, $C_2=2.5$, $L=0.1$ to 1 and $\tau=0.1$

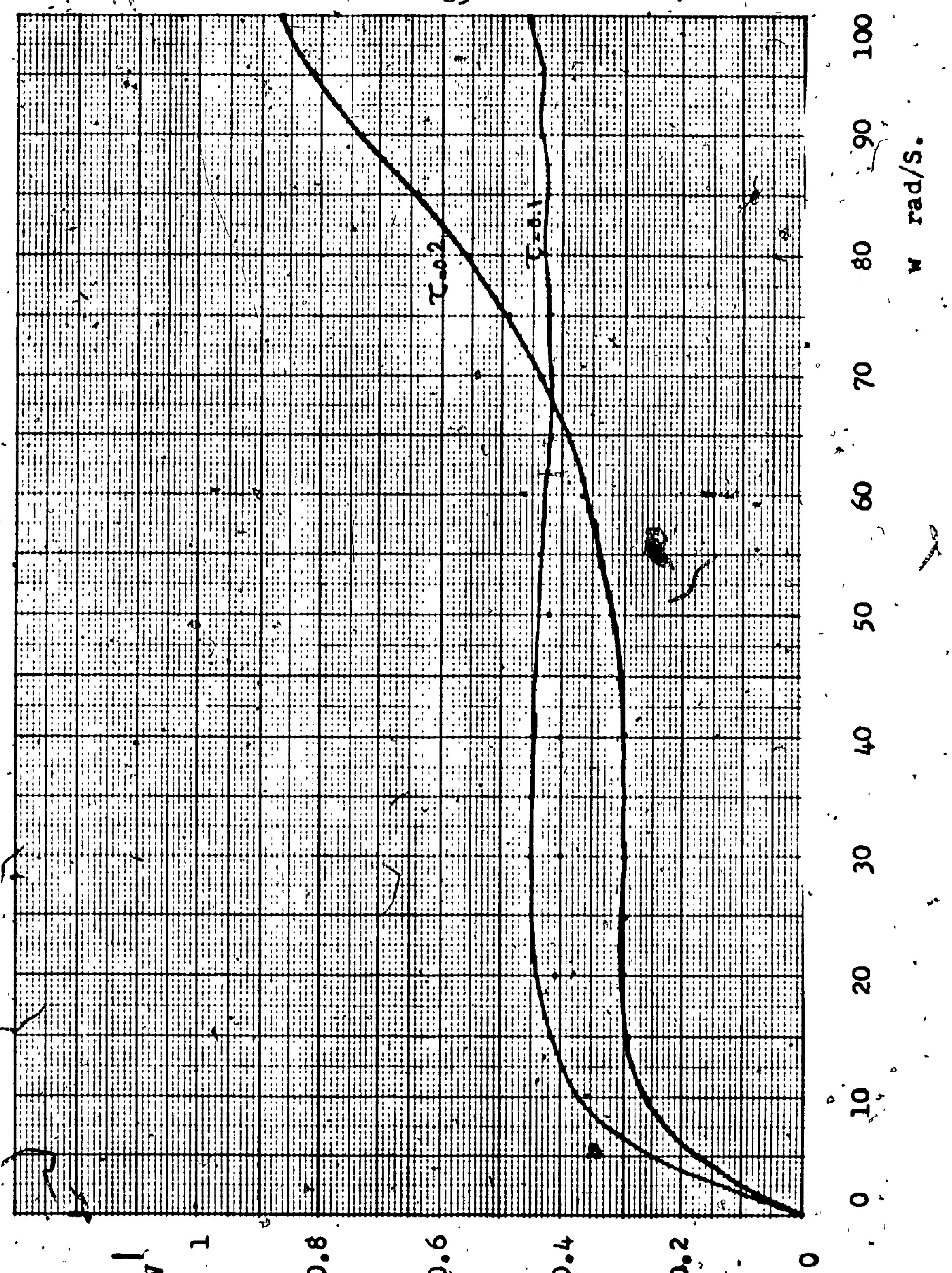

Figure 3.2.12 Response of Independent Elements Type High-Pass Filter.

$C_1=1$, $C_2=2.5$, $L=0.1$ and $\tau=0.1$ to 0.2

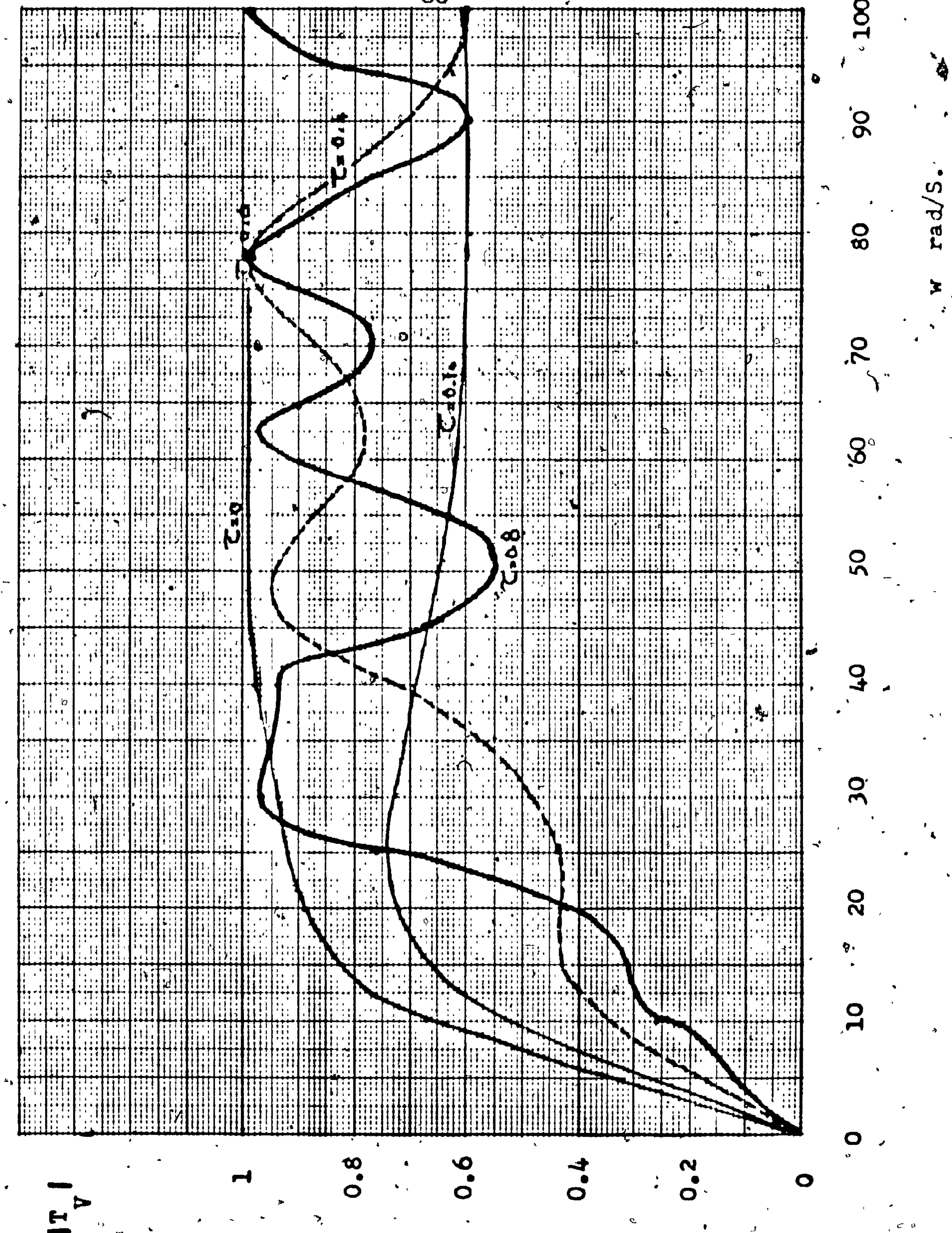

Figure 3.2.13 Response of Independent Elements Type High-Pass Filter.

$C_1=1$, $C_2=2.5$, $L=0.5$ and $\tau=0.1$ to 0.8

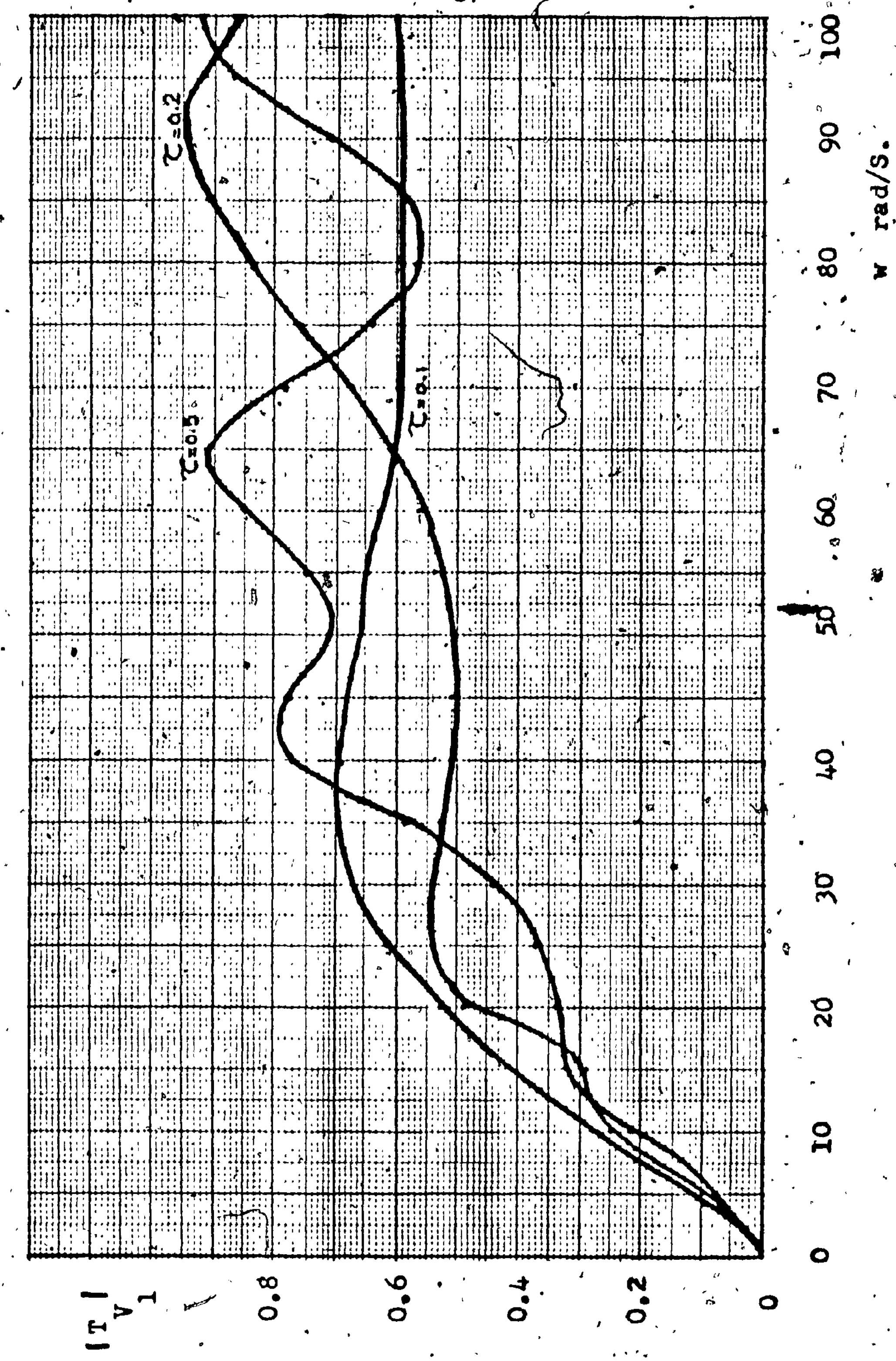

Figure 3.2.14 Response of Independent Elements Type High-Pass Filter.

C_1=0.5, C_2=0.75, L=1.5 and τ=0.1 to 0.5

CHAPTER IV

SUMMARY AND DISCUSSIONS

In this report, we have considered a mixed lumped-distributed filter as shown in figure 2.1, the network consisting of two unit elements (characteristic impedance equal to unity) and three lumped reactive elements, terminated by a one ohm resistor. Both low-pass and high-pass filters are discussed. It is found that in both cases, analytical methods to obtain characteristics prove to be highly difficult. Therefore, computer-aided analysis is to be employed. The cases when the lumped portion corresponds to a constant K filter and a butterworth filter have been discussed in detail. It is shown that for only particular combinations of values of elements and time delay of the unit element give rise to the required characteristics. The general case where all the elements are arbitrary has also been discussed and the computer is so programmed to select these values for which the magnetude of the transfer function is always less than unity. Therefore, it can be concluded that this type of mixed lumped-ditributed filters can be designed with the aid of the computer, as analytical solution is indeed very difficult.

It is suggested that similar studies can be conducted for higher order networks where the lumped sections could constitute any type of filters by the use of computers.

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7. M.N.S. SWAMY AND V. RAMACHANDRAN. "THE A GRAPH OF A CHAIN MATRIX." I.E.E.E. Transactions on Circuit Theory, Vol. C.T-15, pp. 149-151, June 1968